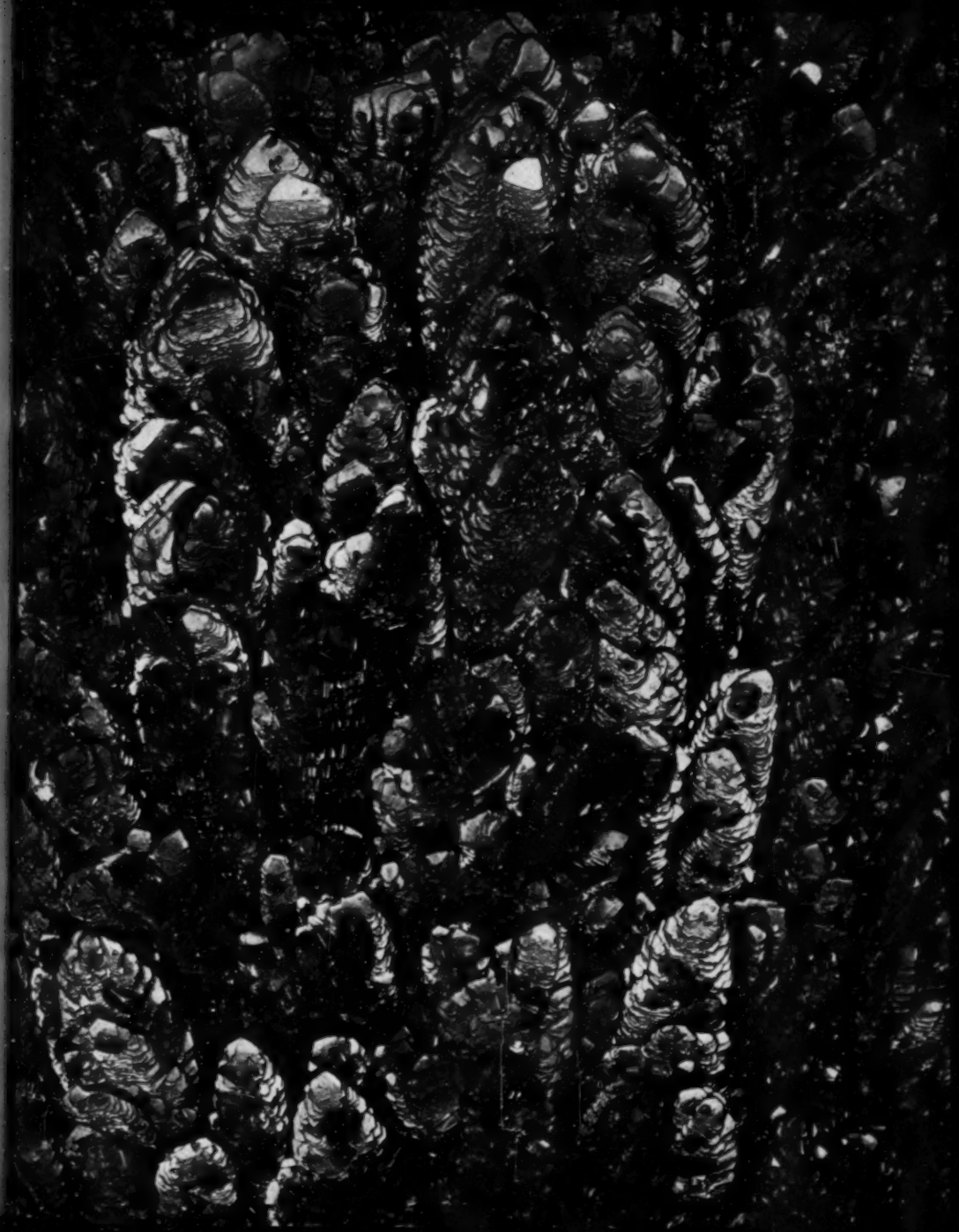
JOURNAL OF APPLIED PHYSICS



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Vol. 14, No. 5 MAY, 1943

In This Issue

Page

Mobilization of Scientific Resources 203

Frictional Phenomena. XIII ANDREW GEMANT 204

Résumé of Recent Research 217

Contributed Original Research

Flow and Recovery Properties of Viscose Rayon Yarn JACK J. PRESS 224

Studies in Lubrication. XI. Temperature Behavior of Journal Bearing Systems M. MUSKAT AND F. MORGAN 234

Fundamental Problem of the Theory of Viscous Fluids HENRI PUTMAN 244

Viscosity of Dilute Solutions of Long-Chain Molecules. V. Dependence on the Solvent MAURICE L. HUGGINS 246

In Every Issue

Calendar of Meetings 217

Here and There 218

New Books 220

Recent Applications of Physics viii

New Instrument Booklets xii

Index to Advertisers xviii

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Mobilization of Scientific Resources

THAT the Office of Scientific Research and Development has accomplished a magnificent job of bringing the scientific talent of the country to bear upon military matters is readily apparent to anyone who has had any dealings with this organization. Under the dynamic leadership of Dr. Vannevar Bush, tremendous strides have been made in the development of new devices and new methods of warfare. Recently, Dr. Bush reported that during the past fiscal year the War Department had placed orders amounting to approximately $560,000,000 for items developed by a single section of OSRD.

It is with considerable surprise, therefore, that one reads in the opening sentences of the bill (S. 702) introduced into the Senate by the Honorable Harley M. Kilgore, Democrat from West Virginia, that "the lack of an adequate appraisal, and the unplanned and improvident training, development, and use, of scientific and technical personnel, resources, and facilities" are "serious impediments" to the effective prosecution of the war. This bill which is called the "Science Mobilization Act" follows closely a similar bill (S. 2721) introduced but not passed during the last session of the Congress. It is one to which all scientists and technical men should pay heed.

The principal objective of this bill seems to be to place the appraisal, coordination, and fostering of science and its training, as well as the promotion of the applications of science into the hands of a powerful government agency to be known as the Office of Scientific and Technical Mobilization. The bill is being introduced as a war measure, but most of its powers are to be applicable in both peace and war. The administration of this Office is to be in the hands of an "Administrator" and a "Board" consisting of one representative each for industry, agriculture, labor, and the consuming public, and *two* members who shall be scientists or technologists!

This bill was introduced into the Senate on February 11 of this year and after two readings was referred to the Committee on Military Affairs. It was introduced into the House of Representatives (H.R. 2100) by Congressman Wright Patman. Hearings were scheduled to begin March 22.

The most important question that arises in one's mind is "Will science under this newly proposed agency develop faster and more usefully than it has in the past few decades under a completely free system in which scientists themselves determine what they shall work on and in which friendly cooperation and competition determine the directions in which science is applied?" Will a board such as that proposed be able to appraise and promote in the embryonic stages such developments as quantum mechanics or artificial radioactivity? These are serious questions which the scientist himself must answer or else he may find them answered by others far less competent.

Frictional Phenomena. XIII

By ANDREW GEMANT

The Detroit Edison Company, Detroit, Michigan

Chapter XIII. Internal Friction in Solids

Abstract

THIS chapter deals with the following topics: (1) The quantitative definition of losses, usually characterized as internal friction losses, that occur in vibrating solids. (2) Experimental methods for the determination of the losses and some of the results obtained on different materials; also a few empirical rules obtainable from the experimental results. (3) The chief mechanisms that lead to internal friction, namely, thermal processes and plastic processes; also the physical basis for the experimentally observed behavior of losses when measured as a function of the frequency of vibrations.

49. General Relations and Definition

All solid materials under a constant load undergo a slow flow process, as discussed in the previous two chapters. The energy expended during this process is to a large extent transformed into heat, only a small portion being stored as elastic energy. If, then, a solid is stressed by an alternating, instead of a steady force, or, expressed more generally, if it undergoes vibrations of any kind, it is to be expected that a certain fraction of its vibrational energy will be continuously transformed into heat. This is, indeed, observed in all cases of vibration, whether under experimental conditions or in practice, and the underlying mechanism is called internal friction of solids.

This sounds very simple in this form, but, as will be shown in this chapter, the problem in reality is more complicated and is not yet fully clarified.

We shall proceed by first presenting the fundamental equations that will allow the internal friction to be defined quantitatively, and thus to be determined by means of measurements.

If a body performs vibrations, the differential equation is of the general form:

$$-m(d^2x/dt^2)-b(dx/dt)-kx+F=0. \quad (140)$$

Here x is the space coordinate that varies with the time t (the deflection of the end of a vibrating beam, for instance); the first term is the inertia term, m depending on the density and a shape factor; the second term is the damping term, to be discussed presently; the third is the elastic term, k depending on the elastic modulus and a shape factor; F is the external force. The first, third, and fourth terms are sufficient wherewith to find the natural frequencies in case of free vibrations and the amplitude *vs.* frequency characteristic in most cases of forced vibrations.

In order to account for two fundamental observations not explained by Eq. (140), if written without the second term, namely, the damping of free vibrations and the finite amplitude in case of resonance for forced vibrations, the second term was introduced into the equation by Voigt. The term implies that the damping force, whatever its nature, is proportional to the first power of the velocity of displacement.

This assumption was introduced because it was in keeping with Newton's original viscosity function, deduced for gases and liquids. This linear relation, introduced in Section 2 and expressed by Eq. (2) and (3), proved, in the light of experiments, to be correct for gases and most liquids. For non-Newtonian liquids (Chapter VIII) and the plastic flow of solids (Chapter XI) it holds only as an approximation for small velocities, the general relation being that of a hyperbolic sine function.

Thus it appears that a term of the form $b(dx/dt)$ cannot hold out much hope for adequately describing the observations, and it will be shown that, indeed, this misgiving is justified. If the term is retained here, it is only on a formal basis. A viscosity coefficient underlying the term $b(dx/dt)$ has no direct physical meaning. However, as has been done previously in this monograph, an equivalent viscosity coefficient, η', will be introduced, one that will satisfy the term $b(dx/dt)$. In the forthcoming discussion (Section 51) the relation of this coefficient η' to

other, more concrete physical concepts underlying the various damping mechanisms will be shown.

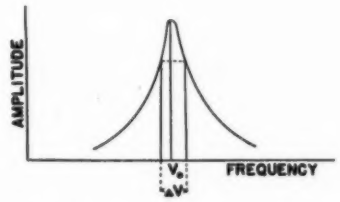

FIG. 108. Definition of resonance bluntness.

The reason that the second term of Eq. (140) is retained here is that the equation then leads to certain expressions that allow some simple quantitative definitions of internal friction; these definitions, in their turn, make it possible to determine the internal friction experimentally. In other words, the concept of η' is retained as a useful mediator between certain physical mechanisms underlying internal friction and some experimentally measurable data referring to internal friction.

It is possible to show on a mathematical basis that retaining the second term in Eq. (140), although fundamentally incorrect, is allowed as a first approximation (compare two related papers by Kimball[1] and Jacobsen[2]). The reason is that the second term is small, as compared with the first and third; in other words damping is relatively small. In consequence, the vibration will remain practically sinusoidal, if the exciting force is sinusoidal, irrespective of whether the damping term is linear or not. Strictly speaking, the vibration will be sinusoidal only if the damping term is linear, but the deviation from the sinusoidal shape will be small if damping is small. Thus it is justifiable to treat the equation as if the damping were linear. It is evident that the factor b will then represent an average value for the particular frequency and amplitude to which the case refers. It is then possible to determine the variations of η' with stress and frequency and to plot η' as a function of these two variables. But it must always be kept in mind that this procedure is only approximate.

As will be shown in this chapter, the physical processes underlying internal friction are not yet definitely established. When they become known, it will be possible, and certainly useful, to write down the rigorous differential equation and arrive at accurate solutions. But as long as the physical process is not fully clarified, the use of such a rigorous method would be illusory; therefore, the use of the approximate method is justified.

The connection between the equivalent viscosity, introduced into Eq. (140), and certain experimentally obtainable data on internal friction will now be shown.

If a body performs free vibrations, Eq. (140) leads to an equation for the amplitude x of the form:

$$x = x_0 e^{-\alpha t} \cos 2\pi \nu_0 t. \tag{141}$$

Here x_0 is the amplitude at the time $t=0$, ν_0 is the natural frequency, and α the damping factor. The decay of the vibration is exponential according to Eq. (141), a conclusion verified by observations. This behavior is only approximate, however, as might be expected from the fact that Eq. (140), underlying Eq. (141), is also an approximation. In reality, α depends upon the amplitude in various ways, but it is still possible to obtain an average value of α from observations of the decay with time. Actual experiments will be quoted below.

Another quantity, the logarithmic decrement δ is often used in preference to α; it is the logarithm of the ratio of amplitudes one period of vibration apart. The period of vibration τ is $1/\nu_0$; hence the ratio of the two amplitudes is $\exp(\alpha/\nu_0)$, and

$$\delta = \alpha/\nu_0. \tag{142}$$

The logarithmic decrement is the most commonly used quantity for the measurement and characterization of internal friction. It has an easily visualizable meaning in connection with heat losses due to internal friction. Since δ is usually

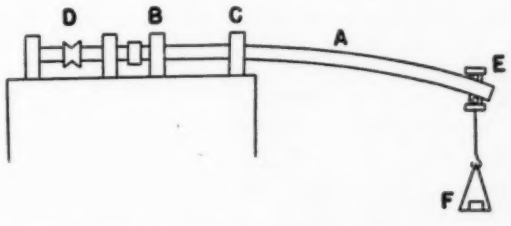

FIG. 109. Revolving rod method for the measurement of internal friction, after Kimball.

a small dimensionless figure, generally (although not always) less than 0.1, it also measures the relative loss of amplitude Δx during one cycle of vibration:

$$\delta = \Delta x/x.$$

Since the energy of vibration is proportional to the square of the amplitude, it follows that the relative loss of vibrational energy ($\Delta W/W$) is given by

$$\Delta W/W = 2\delta. \qquad (143)$$

The energy dissipated per cycle expressed as a fraction of the total vibrational energy is called specific loss, and it is related by means of Eq. (143) to the logarithmic decrement. Thus the latter has an easily visualizable meaning.

The value of α or δ can be computed by using the constants of Eq. (140). The expression obtained by satisfying that equation is

$$\delta = 2\pi^2 \nu b/k, \qquad (144)$$

in which the index of ν is dropped. If actual cases are computed (torsional pendulum, vibrating reed, etc.), it will be found [compare Eq. (131)] that the functions $b(\eta')$ and $k(E)$ are the same. While they are strictly identical in the case of shear, it is more correct to write $3\eta'$ as the argument in case of extension and compression. Keeping this in mind, we will write generally:

$$\eta'/E = b/k.$$

Hence one has for δ:

$$\delta = 2\pi^2 \nu \eta'/E, \qquad (145)$$

an important relation, allowing the computation of the equivalent viscosity from logarithmic

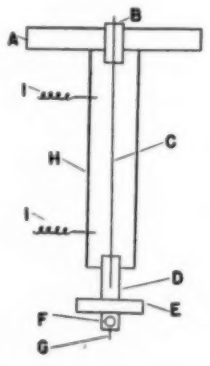

FIG. 110. Apparatus for the measurement of damping of torsional vibrations (Gemant and Jackson).

TABLE XV. Logarithmic decrement δ for several materials, after Kimball.

Material	δ
Phosphor bronze	0.00037
Nickel, cold-rolled	0.0032
Aluminum, cold-rolled	0.0034
Copper, cold-rolled	0.0050
Glass	0.0064
Iron, annealed	0.0079
Wood, maple	0.022
Celluloid	0.045
Rubber, 90% pure	0.26

decrements, obtained from measurements. The physical interpretation of η', as pointed out, will be discussed later in this chapter.

A second fundamental method of determining the internal friction of materials is that of taking the resonance curve under forced vibration. When the exciting frequency equals the natural frequency of the system, the amplitude is kept at a finite level owing to the damping. The amplitude at resonance is given by $F_0/2\pi\nu_0 b$, if F_0 is the amplitude of the external sinusoidal stress. In case there is no damping ($b=0$), this resonance amplitude should be infinite. In determining amplitude and force at resonance, b could be computed. It is easier, however, to keep F_0 constant and vary the frequency, increasing it slightly above and decreasing it slightly below ν_0. The amplitude then drops sharply on both sides, and theory shows that the smaller the damping the sharper will be the drop. The most usual method is to vary the frequency (see Fig. 108) at both sides of ν_0 until the amplitude drops to $1/\sqrt{2}$ of the maximum. The frequency range limited by these two amplitudes divided by the resonance frequency is called the bluntness of resonance $\Delta\nu/\nu_0$, and the experimental determination of this ratio can be carried out conveniently and fairly accurately. Equation (140) then shows that there is a simple relation between this bluntness and the logarithmic decrement, namely,

$$\Delta\nu/\nu_0 = \delta/\pi. \qquad (146)$$

This method thus allows a convenient determination of δ and, by means of Eq. (145), of η'. Later, an experimental method that allows the determination of the bluntness will be described.

It should be added that Eq. (143) is also valid for forced vibration, ΔW signifying the energy

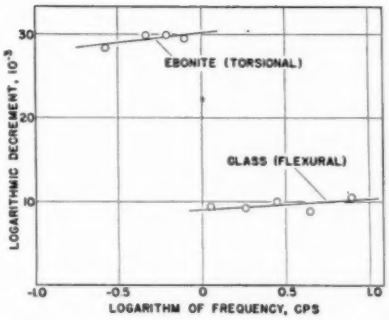

FIG. 111. Logarithmic decrement *vs.* logarithm of frequency for ebonite and glass (after Gemant and Jackson).

lost as heat per cycle. Since the amplitude actually remains constant, the external power source supplies the energy ΔW per cycle required to maintain the vibrations. If ΔW is known, δ and η' can be computed from forced vibrations. Such a case will be considered in the next section.

50. *Methods of Determination and Experimental Results*

In this section three experimental methods will be described that have been used by various authors for the determination of δ, and some of the experimental figures will be presented.

One of the first authors that carried out extensive research in this field is Kimball.[3,4] He used forced vibrations, more specifically the method of the revolving rod (Fig. 109). A is a cylindrical rod of the material to be investigated, supported by bearings B and C and driven by pulley D. By means of the bearing E a weight F is attached to the end, causing a vertical deflection of the rod.

It will be shown in Chapter XIV, in the discussion of shaft whirling, that the internal friction in a rotating and deflected rod produces a force normal to the direction of the deflection, left or right according to the sign of rotation. This force, horizontal in the case shown in Fig. 109, causes a small corresponding horizontal deflection of the end of the rod, until this deflection is compensated for by the tangential component of the weight F. When the pulley D rotates the rod, this rotation is opposed by the constant torque produced by the tangential component of the weight and the energy expended in overcoming this torque is continuously lost as frictional heat. From the measured horizontal deflection this torque, and thus the heat lost per cycle ΔW, can be computed and, by using Eq. (143), δ is found.

In Table XV some of Kimball's data are given which he obtained using frequencies from less than one to several thousands per sec. The figures given are valuable in comparing the different materials listed, but they have not much absolute significance, as the friction coefficient δ depends on a number of factors that influence the structure of the material, as will be seen in Chapter XIV.

It can be seen that dielectric materials have relatively high internal friction, metals having lower values. Among polycrystalline metals, steel has the lowest friction coefficient; among dielectrics, quartz has the lowest.

The chief conclusion Kimball drew from his measurements was that the logarithmic decrement was practically independent of the frequency in the range investigated. We shall come back to this point in discussing the theoretical basis of internal friction.

Next, an experimental procedure, utilizing damping of free vibrations and used by Jackson and the writer,[5] will be described. Two methods were employed, one based on free torsional vibration of shafts, another on flexural vibration of bars. The apparatus used for the first method is shown in Fig. 110. A is a heavy metal support for the chucks B and the shaft C to be tested. By means of the chucks D, a flat weight E is attached to the shaft. F is a mirror and G a steadying pin. H is an electric heating tube, and $I-I$ are thermocouples.

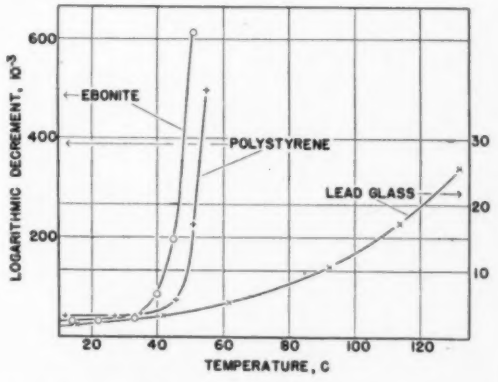

FIG. 112. Logarithmic decrement *vs.* temperature at indicated frequencies for ebonite (0.36–0.26), polystyrene (0.29–0.22), and lead glass (1.68) (after Gemant and Jackson).

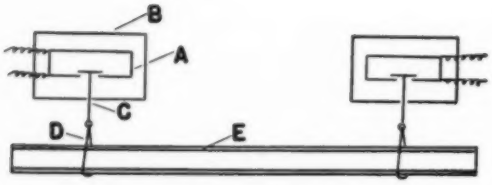

FIG. 113. Diagram of resonance apparatus for the measurement of damping of flexural vibrations, used by Gemant.

Vibrations are excited by means of a bar magnet moved to and fro near a steel pin attached to the disk E; the decay of the vibrations is read by means of a light beam, reflected on F, and a scale. If data at higher temperatures are wanted, the heating tube is used, temperature being measured by means of the thermocouples. The frequency of the free vibrations is varied by changing either the active length of the specimen C or the moment of inertia of the disk E. The apparatus using flexural vibrations is essentially similar.

A question of fundamental importance in all damping measurements is whether the loss measured is due to internal friction of the specimen alone or partly to external losses that are added to the total. Such losses might be caused by motion of the surrounding air or of the solid supports. The former can be eliminated by placing the vibrating specimen into a container with reduced pressure. In Section 9 it has been explained that, the viscosity of gases being essentially independent of density, such a process will not reduce losses due to viscous flow. However, in turbulent flow, the shearing stresses are proportional to the density (see Section 23), thus losses due to turbulent motion will be reduced by such a device. Apparatus like that shown in Fig. 110 can, accordingly, be enclosed in a vacuum, but if no reduction of the damping is observed, such precaution is not necessary. It should be remembered that flexural vibrations are more likely to cause turbulence than torsional ones.

As to the damping caused by the solid supports, there is certainly a component of this type present, and the problem is to minimize it as much as possible. The additional loss due to supports must be smaller than the smallest loss measured; thus, if a material known to have low internal damping shows a low internal friction coefficient when tested in the apparatus, assurance is obtained that the external losses are small. It is further useful to check the apparatus in this respect by making minor adjustments in the support (loosening screws, for instance), or to replace the supporting chucks by others of a different material, and see how those measures affect the observed damping in a given specimen. The design must be such that this component will be a minimum.

Some results of the last quoted authors are shown in Figs. 111 and 112. Figure 111 gives the logarithmic decrement of ebonite and glass at 18°C as a function of frequency. It can be seen that δ is in first approximation independent of the frequency. It must be added, however, that this behavior is not so pronounced in other instances, particularly with metals. Nevertheless, it appears that any theory that requires a pronounced variation of δ with frequency cannot hold generally.

Figure 112 is a plot of the logarithmic decrement *vs.* temperature for ebonite, polystyrene, and lead glass in the temperature range from 10° to 130°C. While δ appears to be constant from 10° to 30°, above 30° it increases more or less rapidly with temperature. It should be added that with soda glass δ decreases near room temperature, there being a minimum in the curve around 70°C.

Numerical data for several materials, as obtained by Gemant and Jackson are given in Table XVI. The frequency range is 0.3 to 10.

TABLE XVI. Logarithmic decrement for several materials (Gemant and Jackson).

Material	Torsion	Bending
Steel	—	1.7×10^{-3}
Copper	—	3.2
Quartz	2.6×10^{-3}	—
Glass: lead	4.2	—
Glass: soft	14	9.5
Wood	—	27
Ebonite	29	85
Polystyrene	48	—

A third method, and, one that is most frequently used for the determination of δ is based on the obtaining of the width of resonance curves, as shown in Fig. 108. It was used by Quimby,[6] Wegel and Walther,[7] Förster,[8] and the writer.[9]

TABLE XVII. Comparison of the logarithmic decrement of various materials.

Material	E (abs.)	δ	$E\delta$
Steel	2000×10^9	0.6×10^{-3}	1.2×10^9
Copper	1250	3.2	4.0
Quartz	350	2.6	0.9
Lead glass	230	4.2	1.0
Wood	130	27	3.5
Ebonite	31	85	2.6
Polystyrene	13	48	0.63
Paraffin wax	5	150	0.75

A schematic diagram of the apparatus as used by the writer is shown in Fig. 113. It consists of two head-phones A, enclosed in shielding iron boxes B, and two needles C, attached to the membranes of the phones. The leads of the left phone connect with a generator, those of the right with an amplifier. Two hooks D made of very thin wire and fixed on the pins C support the cylindrical shaped test specimen E. When the left phone vibrates, the vibrations are transmitted to the specimen, causing it to vibrate flexurally. The amplitude of these vibrations is then measured by means of the right phone and the amplifier. The frequency is varied by adjusting the generator. Whenever one of the natural modes of the specimen is obtained, resonance sets in. The fundamental mode of a hollow cylindrical specimen of outer diameter d_0, inner diameter d_i, Young's modulus E, density ρ, and length l is given by

$$\nu_0=0.88(E/\rho)^{\frac{1}{2}}[(d_0^2+d_i^2)^{\frac{1}{2}}/l^2] \qquad (147)$$

and the succeeding harmonics are obtained by multiplying the right side by 2.76, 5.41, 8.94, 13.3, and so on. It is essential that the position of the hooks D should coincide with the nodes of the vibration, in order to reduce additional losses due to the support.

The method can be used for investigating highly plastic materials, which by themselves would not keep their shape, by enclosing them in a hollow cylinder made of a material of low internal friction, like steel. In such a case the steel would supply the elasticity for the vibrations, and the plastic material, the damping. Heating of the specimen may be effected by enclosing the specimen in an electrically heated oven over the specimen. From the measured decrement δ as obtained from Eq. (146), that of the plastic δ_p can be obtained from the approximate equation

$$\delta_p=(v_sE_s/v_pE_p)(\delta-\delta_s), \qquad (148)$$

where v_s=volume of steel, v_p=volume of the plastic, E_s and E_p=Young's modulus of the steel and of the plastic, and δ_s=logarithmic decrement of steel.

Another example of the resonance method, and one that is more satisfactory for accurate determinations, consists in exciting longitudinal vibrations in the specimen, instead of flexural ones. Transmission of the vibration to and from the test piece is effected either magnetically or piezoelectrically by cementing a small piece of steel or quartz, respectively, to each end of the specimen.

Figure 114 shows some results that the writer obtained by means of the apparatus shown in Fig. 113. The logarithmic decrement of steel and paraffin wax are shown as a function of frequency in the range from 260 to 4600. The values show considerable fluctuation on the average, decreasing slowly for steel and increasing slowly with increasing frequency for paraffin wax.

Decrement data on paraffin wax near its softening point and at a frequency of 260 are shown in Fig. 115. The values were computed from the measured decrement δ by using Eq. (148) and assuming that E_p did not change in the temperature range from 50° to 63°C. The behavior is similar to that of other materials shown in Fig. 112.

In reviewing the material given above, a semiquantitative and only approximate rule can

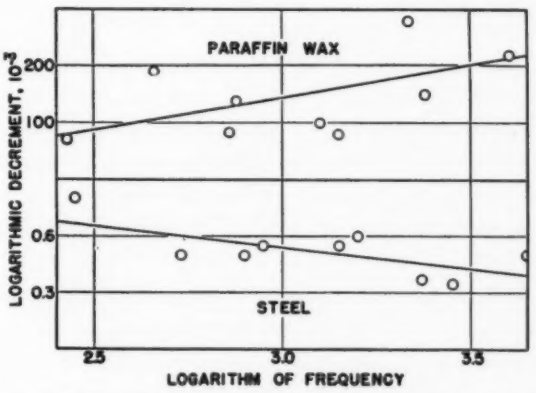

FIG. 114. Logarithmic decrement *vs.* frequency at room temperature for steel and paraffin wax (after Gemant).

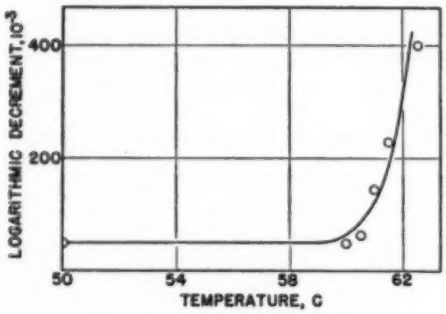

FIG. 115. Logarithmic decrement *vs.* temperature for paraffin wax at a frequency 260 (Gemant).

be established if a comparison of δ of various materials is made (see Table XVII). Here different materials are listed with their elastic moduli and corresponding decrements (both torsional or both flexural). The variation of the elastic modulus is quite large in the whole series, namely, 400 : 1. It is quite striking that δ increases in the same order, the product of the two, sometime referred to as hysteresis constant, being very roughly constant. The "constant" has a value of about 0.7 to 4×10^9, if E is measured in absolute units. The variation of the product is only 6 : 1 in the whole series. The rule can be expressed by the formula:

$$E\delta\sim10^9. \tag{149}$$

It is presented only as a rule of thumb in order to find the order of damping of any material, but not as a generally valid relation. However, correct theories should in some way account for a relation similar to (149).

Measurements of internal friction are complicated by the fact that δ usually is a function of the amplitude of the vibration. It appears that for small amplitudes δ is reasonably constant, and generally increases with increasing amplitude. In particular, metals will exhibit this behavior, as shown in Fig. 116 for two steel samples after the data of v. Heydekampf [10] and Parker.[11]

In summarizing, the following four approximate rules can be derived from the experimental material:

(1) In general, the variation of the logarithmic decrement with the frequency is small.

(2) The decrement usually increases with increasing temperature, slowly at first, then rapidly.

(3) The decrement, usually constant at small amplitudes, increases considerably at larger amplitudes.

(4) The higher the elastic modulus of a material, the lower its decrement.

However, there are many exceptions to these rules. As to rule (1), certain materials show more or less steep peaks at particular frequencies. Such examples will be shown below in connection with Zener's theory. A case where the decrement increases considerably with frequency, was found by Rinehart[12] in the case of lucite (methyl methacrylate polymer), as shown in Fig. 117. The curve refers to a temperature of 26°C, but similar relations were found at −55°, 0°, and 61°C.

Deviations from rule (2) have been mentioned already. Two instances are presented in Fig. 118, one referring to Karolith, a white casein plastic, and another to a soft glass, both exhibiting the minimum mentioned.

The field of single crystals, although highly important from the purely physical standpoint, will not be included in this discussion. Interested readers are referred to the work of Read[13] on single crystals of zinc.

51. Mechanism of Internal Friction

We now turn our attention to the possible mechanisms underlying internal friction. It appears that there are two entirely different processes that lead to losses during mechanical

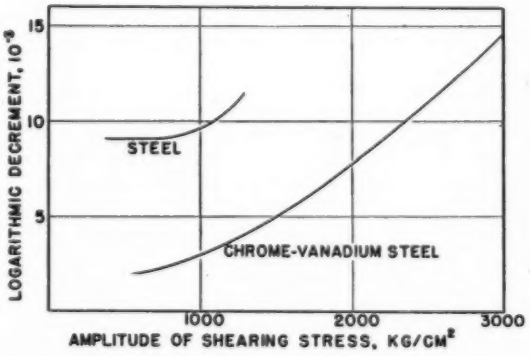

FIG. 116. Logarithmic decrement *vs.* amplitude of shearing stress for steel (Parker) and chrome-vanadium steel (v. Heydekampf).

vibrations. Both appear to be very general and of about equal significance.

One is a thermal process. The possibility of thermal sources of damping has been recognized by several authors, for instance, by Timoshenko,[14] but it was only recently that the idea was put into a more rigorous shape by C. Zener.[15] Zener developed a general theory of thermal losses and considered three different special cases (apart from losses in ferromagnetic materials, the mechanism of which, being too special, should not be discussed here further).

The second process is a strict consequence of the plastic flow of solids, considered in Chapter XI. It is evident, as mentioned in Section 49, that all processes occurring in a unidirectional flow must also occur during vibrations, thus causing corresponding losses. The theory of these losses is not yet developed in a rigorous mathematical form.

Generally speaking, thermal losses will occur in inhomogeneous materials, thus in polycrystalline metals, consisting of a large number of individual grains, probably also in glass, because of irregularly distributed residual stresses. They occur only if the volume elements of the material undergo changes of volume (bending, longitudinal vibrations), but not in the case of pure shear (torsion). As will be shown below, δ according to this mechanism should be independent of the amplitude and exhibit pronounced frequency peaks, dropping to very low values at both sides of a characteristic frequency.

Plastic losses will occur in any material, but presumably most readily in single crystals in which slip occurs easily, and also in materials of

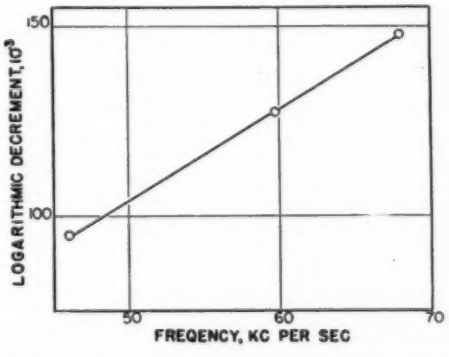

FIG. 117. Logarithmic decrement *vs.* frequency for lucite at 26°C (after Rinehart).

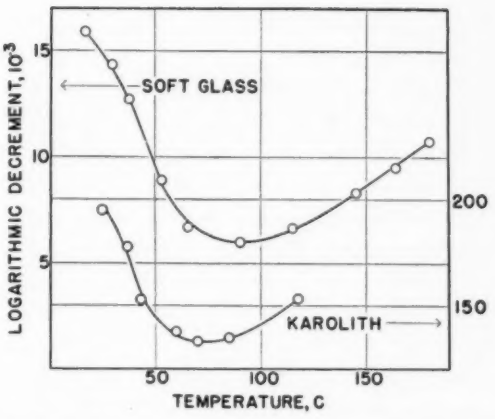

FIG. 118. Logarithmic decrement *vs.* temperature for Karolith, at 59 kc/sec. (Rinehart) and soft glass at 1.64 c/sec. (Gemant and Jackson).

low creep resistance such as lead, among the metals, and the whole group of synthetic plastics. They occur independently of the nature of strain (whether compression or shear). With increasing temperature these losses obviously will be favored [see Eq. (121)], δ also increasing with increasing amplitude [see Eq. (122)]. The frequency dependence of this mechanism is involved; maxima are possible in this case, too, but generally there should be a decrease of δ with increasing frequency.

Whereas in practical cases probably both mechanisms overlap, in scientific research there is a limited possibility of separating the two processes by keeping in mind the factors summarized in Table XVIII.

It is worth noting that neither of the two mechanisms explains the approximate rule (1) mentioned in Section 50, with regard to an approximate independence of δ with frequency, observed frequently by investigators. A probable explanation of this contradiction will be discussed later. We now proceed in picturing briefly both fundamental processes.

(*A*) Zener's *thermal theory* first assumes that the periodic variations of strain cause periodic variations of temperature in the volume elements of the material. This principle evidently holds for all compressional vibrations, since compression causes the temperature to rise and expansion causes it to drop. It is for this reason that the mechanism is inoperative for pure shear for which the temperature remains unaltered. The second

postulate is that—for whatever reason—the temperature variation is different between neighboring volume elements. Thus the temperature will equalize, an essentially irreversible process that must in the final balance lead to a continuous production of thermal energy at the expense of the vibrational energy. While this mechanism is not one of friction in the narrower sense (a force between volume elements exhibiting a velocity gradient), it is friction in the wider sense (transformation of a macroscopic kinetic energy into heat motion).

It can be shown that such a process is effective only in a narrow range of frequency around a characteristic value ν_0, corresponding to a time constant $\tau=1/\nu_0$. If the applied frequency ν is smaller than this characteristic value, heat equalization between volume elements takes place so fast that the temperature gradient always is small and thus the loss itself is small. If, on the other hand, the applied frequency is higher than ν_0, then there is no time for heat flow to take place, and the loss, in spite of the large temperature gradient, is small again. In plotting δ as a function $\nu\tau$, a curve with a fairly sharp maximum is obtained. For the value of δ Zener obtained the following expression:

$$\delta=\frac{\pi\nu\tau}{2TU[1+(\nu\tau)^2]}\int\Delta q\Delta T dv. \qquad (150)$$

Here Δq is the total heat given off by the volume element dv in case of isothermal conditions, and ΔT the temperature rise of the same element for adiabatic conditions, the integral being taken over the whole volume of the material. U is the total vibrational energy. It can be seen that δ is maximum for $\nu=\nu_0$, and falls off to zero toward both sides.

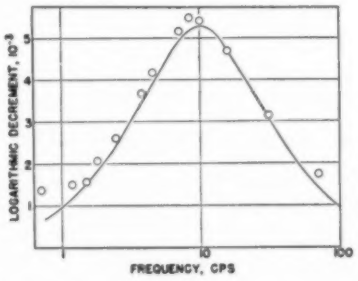

FIG. 119. Logarithmic decrement *vs.* frequency for German silver [measured points after Bennewitz and Rötger, curve computed after Eq. (151) of Zener].

TABLE XVIII. Conditions favoring the predominance of either thermal or plastic losses.

	When thermal losses are to predominate	When plastic losses are to predominate
Strain should be	tension or compression	shear
Structure of material should be	inhomogeneous	homogeneous
Flow resistance of material should be	high	low
Temperature should be	low	high
Amplitude should be	small	large

Three mechanisms are possible that cause the stress, and consequently the temperature rise throughout the material, to vary from point to point. These are as follows:

(1) Stress distribution within the vibrating body.—In the case of flexural and longitudinal vibrations some parts of the specimen are under compression and others are under extension at the same time, hence the temperature gradient. This is the only one of the three cases that allows a rigorous mathematical treatment; Eq. (150) then takes the form

$$\delta=(\pi TE\alpha^2/\rho c)\{\nu\tau/[1+(\nu\tau)^2]\}, \qquad (151)$$

where α=thermal expansion coefficient, ρ=density, c=specific heat. The value of ν_0 and, therefore, τ depends upon the shape of the specimen; for a circular rod it is

$$\nu_0=0.54\kappa/\rho cr^2, \qquad (152)$$

if absolute units are used (κ=heat conductivity, r=radius of cross section).

Figure 119 gives δ for German silver, the measured points after Bennewitz and Rötger,[16] and the curve computed without any arbitrary constant from Eq. (151). The agreement is excellent, but it can be seen how toward both sides the measured values become larger than the calculated ones, probably because of the simultaneous occurrence of other loss processes. The same authors obtained maxima also with glass, steel, aluminum, etc., the position of the maxima agreeing with those computed from Eq. (152). Thus there can be no doubt that this mechanism is one of the loss components, although it might often be masked by others.

(2) Inhomogeneity of the material.—Polycrystalline metals are composed of grains of

different sizes and orientation. Since the elastic moduli are different in different crystallographic directions, the stress in the neighboring grains will vary for a given strain of the material. Thus the heat produced in the different grains will vary, too, causing thermal current of the type considered above.

The smaller the so-called temperature conductivity of the material $\kappa/\rho c$ and the larger the average linear grain dimension a, the larger will be the time constant $1/\nu_0$ of the heat equalization between the grains. Zener deduces

$$\nu_0 = 3\pi\kappa/2\rho c a^2. \tag{153}$$

If for a given material ν is varied, δ should reach a maximum value at $\nu = \nu_0$. The same effect is obtained if the frequency is kept constant and the grain size varied, the curve in such a case going through a maximum when the value of a fulfills Eq. (153). The grain size at which the maximum occurs should increase with decreasing frequency.

These conclusions were verified by experimental work of Zener *et al.*[17] on brass of varying grain size. Figure 120 shows the results for two different frequencies: 36 and 12 kc. The arrows indicate the position of maxima, computed from Eq. (153). The agreement is good. We shall come back to this mechanism in the next chapter.

(3) Residual stresses in the material.—Even a homogeneous material might exhibit an effect similar to that discussed under (2) if a residual internal stress, varying from place to place, were present in the material. In case of an external stress the total stress then will vary from point to point, and this again will lead to local thermal currents.

(*B*) Concerning the second fundamental process, *plastic mechanism* of internal friction, it is evident—as pointed out at the beginning of this chapter—that a plastic flow accompanying elastic displacements must, in case of vibrations, lead to damping. This conclusion is in line with the general statements made in Chapter I, Section 2. Also the effect can easily be visualized by considering the so-called hysteresis loop. By hysteresis is understood the fact that the strain in solids is not a function of the momentary stress alone but also of those immediately preceding, this being true if the solid exhibits plastic flow.

Figure 121 shows the stress-strain diagram of a vibrating solid. The straight line AB represents a vibration at small amplitudes at which, as shown in Chapter XI, plastic flow is negligible. Here a single strain belongs to any stress, and there is no hysteresis. For larger amplitudes the closed curve $CDEF$, which is run through in clockwise direction, represents the vibration cycle. There are two different strains for any value of the stress, because of the permanent plastic deformations. Thus, for stress zero at C there is a residual strain OC from the previous cycle, and after another half-cycle when the stress becomes zero again, the residual strain is OE. In moving along the element GH, the shaded area represents the element of work the external stress has to perform on the solid. The total work done during one complete cycle is obtained by integration and the latter obviously yields the area of the loop. The reason is that the work during CD is larger than the energy regained during DE, because of the additional stress necessary to annihilate the residual strain OC.

It is easy to show, however, that the steady-state flow resistance of solids cannot account quantitatively for the observed losses. The logarithmic decrement δ of a solid having elasticity E and flow resistance η is given by an equation analogous to (145), namely,

$$\delta = E/2\nu\eta. \tag{154}$$

Now, the order of magnitude for E is 10^{11}, that of ν, say, 10^3, and that of η around 10^{16}. That gives for δ the order 10^{-8}, while the order as

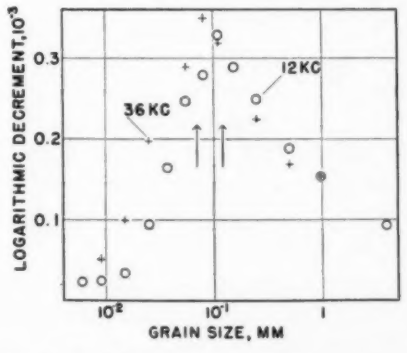

FIG. 120. Logarithmic decrement *vs.* grain size at two different frequencies for brass (after Randall, Rose, and Zener).

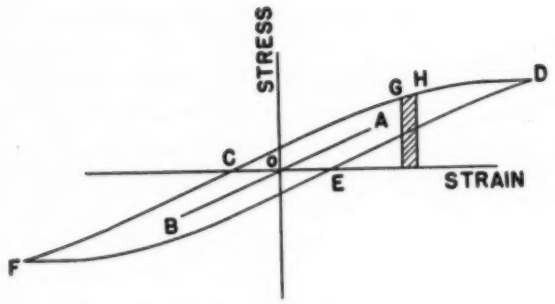

FIG. 121. Hysteresis loop.

observed experimentally is 10^{-3}, an obvious disagreement.

This disagreement disappears, however, if one recalls that the initial process, whether of a slip or a creep, is several orders faster than the steady rate part, and the initial flow resistance accordingly several orders smaller than the steady rate resistance. While a rigorous theory of damping based on this mechanism cannot be developed until we know more about the initial process, a rough consideration might be of some help. This will be presented briefly, together with an experimental verification of the theory based on data obtained on lead.

It should be recalled that a liquid possessing viscosity and elasticity was represented by a model shown in Fig. 57. The same model holds for a vibrating solid having plasticity and elasticity. As just mentioned, damping will probably be determined by the initial displacements, and so an element corresponding to this initial process will have to be added to the previous model. This is shown in Fig. 122(a). Here the dashpot η represents the steady state flow resistance and the spring E the elastic modulus. While the displacement of E is instantaneous, the initial process can be represented by the series spring E^*, having the dashpot η^* in parallel. When E^* is strained according to the stress acting on the system, this transient part of the energy-consuming process will stop and the one displacement, which progresses slowly with time, is that of the dashpot η.

It cannot be emphasized strongly enough that such a presentation is very crude, since the physical processes are certainly much too complicated for allowing a presentation by means of simple lumped elements. Figure 122(a) would indicate that the relaxation process is an exponential. The more detailed Prandtl model (Section 42), although it correctly shows that this is not the case, is still far from a physical reality. The purpose of the figure is to give a rough idea of the losses that can be expected in a vibrating plastic solid.

Figure 122(b) is the equivalent model underlying Eq. (140) having elasticity E' and equivalent viscosity η'. Equation (145) establishes the connection between the decrement δ and this viscosity η'. If, now, diagram (b) can be computed from (a), then the decrement δ can be computed from the measurable relaxation components E^* and η^*.

The equations that will transform diagram Fig. 122(a) into (b) are comparatively simple.[18] The complete set will not be reproduced here. Generally δ will go through a maximum with increasing frequency, the critical frequency being given by the ratio E^*/η^*, the reciprocal of the time constant of the relaxation element in model (a). It is to be expected that this frequency will be anywhere in the low frequency range (from fractions of 1 to 10^3 c.p.s.). In the frequency range above this critical value approximate formulas will hold, namely,

$$E' = E \tag{155}$$

and (with $\omega = 2\pi\nu$)

$$\eta' = E^2/\eta^*\omega^2. \tag{156}$$

From the latter we have:

$$\delta = E/2\nu\eta^*. \tag{157}$$

Interesting conclusions can be drawn from these relations. Equation (155) shows that the order of magnitude of the quantities involved is often such that E^* will not affect the value of E', the latter being simply given by the modulus E, as measurable from static experiments. This is in agreement with vibration tests, often yielding a modulus not too different from that known from static tests.

The second conclusion relates to the decrement δ. It can be seen that formula (154), valid for a Maxwellian body (spring and dashpot in series), will hold for the vibrating body above the critical frequency, with the relaxation resistance as the resistance of the dashpot. This explains im-

mediately why Eq. (154), for which the steady state resistance η was used, yielded wrong values for δ. With the order of magnitude of η^* the computed value of δ will be several orders higher, in agreement with measurements.

Experimental figures of Lyons[19] on lead will now be used to show the validity of Eq. (157). Figure 123 gives Lyons' curve on the initial compression of lead *vs.* time at a stress of 121 kg/cm². It can be seen that it is not an exponential curve, but the dotted curve was drawn as an approximation, having the same final value and the same initial slope as the experimental curve. The time constant of this curve is 0.08.

From the known stress and the final deformation (0.051), E^* was computed to be 2.4×10^9 abs. and from the time constant 0.08, $\eta^*=0.19\times10^9$ abs. was obtained. In order to complete the values of the model elements for lead, that of E (0.16×10^{12}) and that of η (100×10^{12}) should be noted, the latter obtained from the long-time compression tests of Lyons (see Section 47).

With these values then it can be verified, first, that for frequencies >10 c.p.s. the use of the simplified equations (155) to (157) is justified. Using the last named equation $\delta=0.42$ is obtained for $\nu=10^3$ and $\delta=0.042$ for $\nu=10^4$.

While experimental values very seldom exhibit a strong variation of δ with frequency, a fact mentioned in connection with Zener's theory, the experimental figures in the above frequency range agree well with the computed order of magnitude. Measurements of Wegel and Walther on lead in the frequency range 10^3-10^4 indicate a slight decrease of δ with increasing frequency, the measured value for the so-called hysteresis

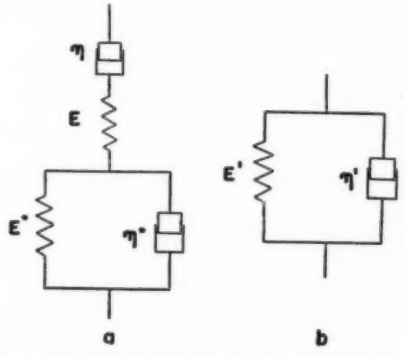

FIG. 122. Equivalent models, using lumped elements, of a vibrating solid. (a) Model based on plastic properties of solids. (b) Model underlying Eq. (140).

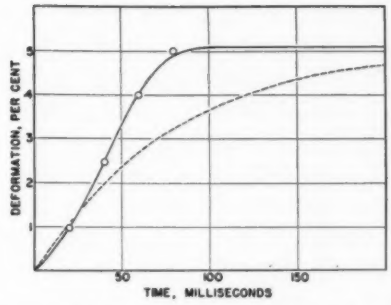

FIG. 123. Initial compression of lead, according to Lyons. Dotted line is an exponential having same initial slope and same final value as the experimental curve.

constant ($E\delta$) being about 15×10^9. Thus for δ a value of around 0.1 was obtained experimentally, the order of which agrees well with the two values computed above.

The above considerations thus show how the initial plastic processes necessarily must lead to relatively high losses in case of vibrations. More experimental data on relaxation of other materials in the time range of less than 1 sec. will be necessary for a further check of this mechanism.

There is one important question that deserves additional discussion; this is that experimental δ *vs.* frequency curves are as a rule much flatter than is required by either theory. Two explanations can be offered, one holding for both thermal and plastic losses, the other for plastic losses in particular. These are as follows:

(1) One possible explanation that is generally assumed is that in inhomogeneous materials one cannot operate with a single mechanism having a single time constant. Usually there will be several parallel processes, each with its own time constant, and if the decrement for such a material is calculated, curves will be obtained that are quite flat if there is a large number of time constants.

Looking into the two fundamental processes described under *A* and *B*, above, one will have to admit that, indeed, single time constants appear unlikely. As an example, consider Eq. (153). Here a was described as the average grain size. However, it is more likely that grains of a different size will determine different time constants, and the individual peaks, as independent of each other, will then have to be added.

Materials that approach an ideal simple structure should exhibit sharp peaks; materials

that are used in technical applications and are of a more or less heterogeneous structure should show rather flat δ-frequency curves, as is indeed the case. The trouble is that an experimental verification of theoretical equations becomes more and more difficult as the materials deviate from simple structures.

(2) In the approximate theory of plastic losses, as outlined above, one essential point was disregarded for sake of simplicity, and it is possible that this point has a considerable influence upon the final results. This is the decrease of the flow resistance with increasing rate of deformation, considered in detail in Chapters XI and XII. Strictly speaking, such resistances would prohibit the use of simple equivalent models such as that given in Fig. 122. However, as an approximation, it is possible to operate with that model and the equations deduced therefrom, and afterwards to consider the variability of the resistance.

Thus it should be assumed that the rate of deformation D of the element η^* increases exponentially with stress S:

$$D \sim \exp S.$$

The ratio D/S, proportional to $1/\eta^*$, is then:

$$1/\eta^* \sim D/\log D \sim D,$$

since the variation of $\log D$ can be neglected as compared with that of D itself. In case of sinusoidal vibrations we have

$$D \sim \nu A,$$

if A is the amplitude of the deformation. Thus:

$$1/\eta^* \sim \nu A.$$

If this value is substituted into Eq. (157), we obtain

$$\delta \sim A, \tag{158}$$

the proportionality constant depending only upon the material.

Equation (158) contains two results. First, the rapid variation of δ with frequency disappeared. Obviously, this should not be taken literally, the deduction given being much too rough. But it can be seen that a consideration of the stress dependence of the resistance tends to flatten out the δ-frequency curves.

The second result is that the decrement increases with the amplitude of vibrations. This is completely in keeping with observations on vibrations with larger amplitudes. Again, the real dependence will not be linear, as suggested by Eq. (158); all we might say is that one will increase with the other.

Bibliography to Chapter XIII

1. A. L. Kimball, Trans. A. S. M. E. **51**, 21 (1929).
2. L. S. Jacobsen, Trans. A. S. M. E. **52**, 15 (1930).
3. A. L. Kimball, *Vibration Prevention in Engineering* (Wiley and Sons, New York, 1932).
4. A. L. Kimball, J. App. Mech. **8**, A-37, A-135 (1941).
5. A. Gemant and W. Jackson, Phil. Mag. **23**, 960 (1937).
6. S. L. Quimby, Phys. Rev. **25**, 558 (1925).
7. R. L. Wegel and H. Walther, J. App. Phys. **6**, 141 (1935).
8. F. Förster and W. Köster, J. Inst. Elec. Eng. **84**, 558 (1939).
9. Andrew Gemant, J. App. Phys. **11**, 647 (1940).
10. G. S. v. Heydekampf, Proc. A. S. T. M. **31**, 157 (1931).
11. E. R. Parker, Trans. Am. Soc. Metals **28**, 661 (1940).
12. J. S. Rinehart, J. App. Phys. **12**, 811 (1941).
13. T. A. Read, Phys. Rev. **58**, 371 (1940).
14. S. Timoshenko, *Strength of Materials* (D. van Nostrand, New York, 1930), Vol. 2, p. 675.
15. C. Zener, Proc. Phys. Soc. **52**, 152 (1940).
16. K. Bennewitz and H. Rötger, Zeits. f. tech. Physik **19**, 521 (1938).
17. R. H. Randall, F. C. Rose, and C. Zener, Phys. Rev. **56**, 343 (1939).
18. F. A. Firestone, J. App. Phys. **9**, 373 (1938).
19. W. J. Lyons, J. App. Phys. **11**, 514 (1940).

Résumé of Recent Research

Diffusion of Carbon in Iron Alloys

One of the most fundamental mechanisms of various technically important changes occurring in metals and alloys is diffusion. In particular the diffusion of carbon in iron and its dependence upon temperature is responsible for the great variety of properties obtained in heat treatment of steels. Many of these properties are influenced by addition of various alloying elements which also affect the rate of the reactions occurring in these systems. R. Smoluchowski has recently studied the influence of cobalt,[1] tungsten, and molybdenum[2] on the diffusion rate of carbon in iron. The specimens, one containing carbon, the other without carbon, were welded together and then kept for three days at 1000°C in a quartz tube filled with argon. Subsequently, thin layers were turned off parallel to the interface and their carbon content determined. From the concentration distribution in each sample the coefficient of diffusion D could be determined.

The results of the experiments with various iron alloys are shown in Fig. 1 where the diffusion coefficient of carbon in iron is plotted against the atomic percentage of the added element. Cobalt accelerates the diffusion of carbon very strongly —even more than carbon itself. This is particularly interesting because cobalt leaves the iron lattice practically unchanged at these concentrations; carbon, on the contrary, expands it rather rapidly due to its interstitial location. Nickel[3] and manganese[3] have almost no influence on the diffusion of carbon although each affects the crystal lattice in a different way. Tungsten and molybdenum[4] increase the lattice constant of iron very similarly, but they slow down the diffusion of carbon in quite a different degree. All this applies of course at 1000°C where the measurements were performed.

These experiments indicate that the change of the diffusion rate of carbon in iron cannot be treated as a simple geometrical problem of change of lattice parameter; the influence of the alloying elements on the electronic interactions between carbon and iron atoms has to be taken into account.

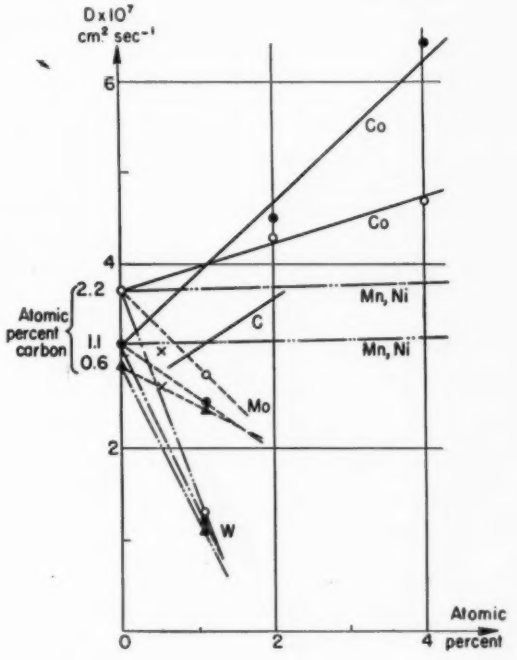

FIG. 1. Diffusion coefficients of carbon in various iron alloys at 1000°C. (Data for Mn and Ni were obtained from reference 3; crosses indicate values for Mo based on reference 4.)

[1] R. Smoluchowski, Phys. Rev. **62**, 539 (1942).
[2] R. Smoluchowski, Phys. Rev. (in course of publication).
[3] C. Wells and R. F. Mehl, Trans. A. I. M. E. **140**, 279 (1940).
[4] J. L. Ham, R. M. Parke, and A. J. Herzig, American Society for Metals, Preprint, 1942.

Calendar of Meetings

May
14–15 Acoustical Society of America, New York, New York

June
14–16 American Society of Mechanical Engineers, Los Angeles, California
14–19 American Association for the Advancement of Science, Corvallis, Oregon
14–19 American Association of Physics Teachers, Corvallis, Oregon
16–17 American Mathematical Society, Corvallis, Oregon
17–19 American Physical Society, State College, Pennsylvania
18–20 American Association of Physics Teachers, Chicago, Illinois
18–20 Society for the Promotion of Engineering Education, Chicago, Illinois
21–25 American Institute of Electrical Engineers, Cleveland, Ohio
28–July 2 American Society for Testing Materials, Pittsburgh, Pennsylvania

July
10 American Physical Society, Stanford University, California

Here and There

Cover Photograph

The cover photograph shows crystals of metallic magnesium which have been produced by the dolomite ferro-silicon process by the National Research Corporation, Boston, Massachusetts. This process has demonstrated the possibility of employing relatively high vacuum (that is pressures of 10^{-5} atmosphere and less) on an industrial scale.

★

News of Societies

The Electrochemical Society met at the Hotel Roosevelt in Pittsburgh on April 8, 9, and 10. Scientific-technical sessions during the three-day period were concerned principally with subjects bearing on the war effort. The subjects under discussion were: Automatic Control, Dielectrics, and Corrosion. Following an address by Professor Edwin M. Baker, retiring President of the Society, the Young Author's Prize and Book Prize were awarded to Dr. Sidney Speil, Associate Non-Metals Engineer at the Electrotechnical Laboratory, U. S. Bureau of Mines, Norris, Tennessee. Mr. George E. Pendray, Assistant to the President of Westinghouse Company, addressed the Society on "Rockets."

The Society of Rheology and the Polytechnic Institute of Brooklyn held a joint research conference on "The Present State of the Kinetic Theory of Rubber Elasticity" on Saturday, April 3, at 10:30 A.M. at the Institute. The program included papers by Dr. M. L. Huggins, of the Eastman Kodak Company Research Laboratory, Rochester, New York, on "Statistical Treatment of Long Chain Molecules," and by W. L. Wood, of the National Bureau of Standards, on "The Present Experimental Aspect of Rubber Elasticity." These papers were discussed by authorities on the subjects.

The election of officers of the American Association for the Advancement of Science for 1943 was delayed because of cancellation of the New York meeting. However, an election has been held by mail ballot of the council, a procedure that is valid under the constitution of the association. J. W. Beams, University of Virginia, was elected Vice President of Physics (Section B) and Elmer Hutchisson, New York City, was elected section committeeman of the Physics Section for a term of four years, expiring at the close of the meeting of December-January, 1946–47.

The Society of the Sigma Xi installed its 86th chapter at the Polytechnic Institute of Brooklyn, Brooklyn, New York on Thursday, March 25, 1943. The installing officers were Dr. Harlow Shapley of Harvard University, National President, and Dr. George Baitsell of Yale University, National Secretary. Twenty-six chapters of the society were represented by delegates at the installation, which was preceded by a convocation where Dr. Harry S. Rogers, President of the Polytechnic, greeted the delegates and spoke on the subject "Education and Research." Dr. Shapley responded with an address entitled "On the Hitching of Wagons." The new chapter at the Polytechnic has twenty-five charter members. Officers of the local society elected at the installation included: Professor Clyde C. Whipple, President, Dr. Raymond T. Ellickson, Secretary, and Dr. Chilton A. Wright, Treasurer.

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Staff Changes

Roy S. Laird, Sales Manager of the Ohmite Manufacturing Company, Chicago, has been named Vice President of the company.

Carl H. Flink, whose appointment as Technical Secretary of the American Society of Heating and Ventilation Engineers has recently been announced, will assist in the compilation of codes and standards and work closely with the Guide Publication Committee in the preparation of the Society's annual reference book. Mr. Flink has had positions as Chief Engineer of the Institute of Thermal Research of the American Radiator Company, Director of Research for the American Gas Products Corporation, and Mechanical Engineer in Charge of Product Development Laboratory of the American Radiator and Standard Sanitary Corporation. During the first World War he served with the Meteorological Division of the U. S. Army Signal Corps.

Roy M. Smith who joined the Roller-Smith Company, Bethlehem, Pennsylvania, in August, 1942, as Assistant Chief Engineer has been appointed Chief Engineer, succeeding J. D. Wood, resigned. Mr. Smith has formerly been Engineering Manager for the Wiring Device Division of the Bryant Electric Company and Section Engineer on Relay Design and Application for the Westinghouse Electric and Manufacturing Company.

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Honors and Awards

James L. McCarthy, engineering representative for the Westinghouse Electric and Manufacturing Company at Honolulu was voted the Order of Merit, highest company award for outstanding service, by the Westinghouse Board of Directors: Mr. McCarthy was recently commended by the Navy for excellence of performance in assistance rendered to ships and engineers at Pearl Harbor.

Professor Arthur B. Lamb, Dean of the Graduate School of Arts and Sciences at Harvard University, has been awarded the 1943 William H. Nichols Medal of the New York Section of the American Chemical Society. In making this award the Society praised Professor Lamb for "his investigations in inorganic and physical chemistry, leadership in defense against poison gas, and as a teacher, administrator, and editor." During the first World War, Professor Lamb was a lieutenant colonel in the Research Division of the Chemical Warfare Service.

The medal of the British Institute of Metals has been presented to a former president of the Institute, Dr. Harold Moore, C.B.E., Director of the British Non-Ferrous Metals Research Association, a leader in industrial research in England. The medal is unique in that it is made of pure platinum.

The Council of the Physical Society, London, has awarded the Charles Chree Medal and Prize for 1943 to Professor (now Colonel) B. F. J. Schonland and the Duddell Medal for 1942 to Dr. C. R. Burch. Colonel Schonland, formerly Professor of Physics at Cape Town and afterwards Director of the Bernard Price Institute of Geophysics at Johannesburg, is now in Great Britain engaged in scientific work in connection with the war. Dr. Burch, formerly of the Research Department of the Metropolitan Vickers Electrical Company, has continued his work in the physics laboratories of the Imperial College and the University of Bristol.—*Science*, March 19, 1943, Vol. 97, No. 2516, p. 260.

Dr. Irving Langmuir, Associate Director of the General Electric Research Laboratory, Schenectady, has been elected to honorary membership in the Institute of Metals, of London. Notification of this honor was given in a letter just received by Dr. Langmuir, winner of the 1932 Nobel Prize in Chemistry, from the Institute's secretary, G. Shaw Scott.

Though Dr. Langmuir has not recently engaged directly in the science of metallurgy, his work has had considerable indirect effect. His investigations on surface films have been applied to great advantage in connection with the flotation process used in separating metals from ores. His point of view as to the behavior of atoms and molecules on surfaces has proved helpful in the study of metals. He has worked on the emission of electrons from metals when heated. This is at the basis of many modern applications of electronics, such as radio vacuum tubes and has also led to a better understanding of the nature of metals. He studied the evaporation of metals in a vacuum, an effect important in connection with the tungsten lamp. Other researches have concerned the effects of gases in metals, of which the importance has only recently been recognized.

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Research Assistantship

A research assistantship in petroleum physics has been established by the Gulf Oil Corporation at the Massachusetts Institute of Technology. The recipient must qualify for a tuition scholarship and be a candidate for an advanced degree in the Department of Physics. The stipend will be $1000 per school year, subject to renewal at the end of nine months under an accelerated program. Further information may be obtained from the Admissions Office of Massachusetts Institute of Technology.

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Necrology

Henry Kasner, for thirty years associated with the Radio Corporation of America and its predecessor, the American Marconi Company, died February 14 at the age of 53.

Arthur Livingstone Kimball, research physicist and consulting engineer for the General Electric Company, and a specialist in vibration problems, died March 20 at the age of 57.

Dr. Frank Sturdy Sinnatt, Director of the Fuel Research Station of the British Department of Scientific and Industrial Research, died on January 27 at the age of 62.

Rollo Appleyard, cable engineer, physicist, and inventor, died on March 1 at the age of 76.

New Books

Hardness and Hardness Measurements

By Samuel R. Williams. Pp. 558+vii, 16×23½ cm. American Society of Metals, Cleveland, Ohio, 1942. Price $7.50.

The purpose of this book is to undertake a scholarly review of the history and methods of hardness measurement. Although the book is published in the well-known metallurgical series, Professor Williams does not attempt to restrict the discussion to the determination of the hardness of metals, but includes a great deal of information on the hardness of substances such as glasses and abrasives.

It should be emphasized that the book is essentially a popular one in the sense that it will be of most interest either to an expert who reads it for its historical content or to a beginner who would find the going hard in a really technical book and desires a qualitative survey of the entire field rather than a quantitative treatment of the three or four best-known methods of determining the hardness of materials. To counterbalance this aspect of the book, the author has a very thorough bibliography which is carefully edited and compartmentallized.

The first two chapters of the book deal with the theory of matter, in a purely descriptive way, with the intention of showing that there is a correlation between atoms and the physical world of everyday life. This part is frankly a great disappointment to the reviewer. Although the constituent particles of the atom from the electron to the neutron are mentioned (with a bit of cosmic-ray physics and positron theory thrown in for good measure), there seems to be no reference to the many fine books and review articles which have been written by people in and out of industrial laboratories in a desire to show what modern physics really has to say about the relation between the atom and the properties of solids. If this type of work is not mentioned in books that are written for metallurgists by physicists, it is difficult to see just how the more recent contributions of physicists to the underlying principles of metallurgy will be made known where they can be of value.

Following the introduction, are a number of chapters which make up the core of the book and deal with the historical and the modern development of each of several types of hardness measuring methods. In sequence these are as follows: Scratch method, indentation method (separate chapters on the Rockwell method, the Brinell method, and various pyramidal-indenter methods), and the rebound method. This part of the book is excellent, for the historical parts are interesting to read and the modern parts are both comprehensive and profusely illustrated. This is followed by specialized chapters dealing with a detailed qualitative description of the flow of metal near the indentation in the various methods and with the application of hardness measurements to specialized fields outside metallurgy, ceramics, and mineralogy, such as the determination of the hardness of wood, wax, rubber, and so forth.

The last chapters of the book deal with a summary of the correlation between hardness measurements and other properties of materials such as electrical and magnetic properties. These are not profound, but should be of interest to the working metallurgist who is always interested in knowing what things hardness tells him.

On the whole, the reviewer heartily endorses this book for its primary purpose, namely, to survey the broad field of hardness measurement. Any person interested in hardness measurement will find it a worth while addition to his bookshelf.

Frederick Seitz
Carnegie Institute of Technology

The Oceans

By H. U. Sverdrup, Martin W. Johnson, and Richard Fleming. Pp. 1087+x, 15½×23½ cm. Prentice-Hall, Inc., New York, 1942. Price $10.00.

This stimulating volume presents in summarized form the whole of present oceanographic knowledge. Its full title, *The Oceans, Their Physics, Chemistry, and General Biology*, indicates the inclusive nature of the work and suggests the large amount of factual material packed into the book's 1087 pages, 126 tables, and 272 figures and charts. The three authors, all members of the staff of the Scripps Institution of Oceanography, have combined their talents to present all phases of oceanography.

The subject matter presented may be divided into three major sections. The first includes chapters on the physical and chemical properties of sea water, the topography of the ocean floor, and the types of animal and vegetable life present in the oceans—in other words, purely descriptive discussions of the basic factors in the sea.

Secondly, a group of chapters outlines the science of physical oceanography; included here are such subjects as the temperature distribution in the sea, ocean currents, tides, and waves. This section is more nearly analytical and deductive than the other two; except for the classical theory of waves and tides, however, physical oceanography has not yet been sufficiently studied to indicate clearly the factors responsible for the physical equilibrium of the sea. This is noticeably true, for example, in most of the discussion of ocean currents.

The third and final section treats the biological equilibrium of the oceans. This naturally qualitative discussion explains many of the mutual interrelations between plants, animals, and the physico-chemical environment, but raises even more problems that are unsolved.

This survey of oceanography as a whole should help to set different problems in their proper perspective and will doubtless be most stimulating to future research. The book itself, with its extensive bibliographies following each

chapter, and thorough subject and name indices, should be for some time a required reference work for all students of oceanography. To the general scientific reader, the volume will be of interest partly for the wealth of factual information it contains, and partly for the emphasis it places on the development of an adequate theory to explain the manifold interrelations of oceanic phenomena. Such a theory is still in its infancy, but *The Oceans* should do much to advance its growth.

LYMAN SPITZER, JR.
Columbia University
Division of War Research

Heat

BY JAMES M. CORK. Second edition. Pp. 294+xiii, Figs. 128, 16×23½ cm. John Wiley and Sons, Inc., New York, 1942. Price $3.50.

For ten years Cork's textbook on heat has been of value to teachers and students of intermediate physics by bridging the gap between an advanced, lengthy treatise and the simplified, brief account given in the average textbook of college physics. The present volume is the second edition and covers the same material in very much the same way as the first edition. The general plan of the book has been improved by condensing what was formerly two chapters entitled "Change of state" and "Continuity of state," into one chapter called "State of matter," and in treating the production of both high and low temperatures in one chapter.

Several new topics have been added, among which are to be found: a history and description of fluid thermometers, the international temperature scale, the measurement of very low temperatures, the measurement of the viscosity of a gas, vibratory method of measuring the ratio of the specific heats of a gas, the experimental determination of the ratio of thermal to electrical conductivity, the absolute measurement of electrical resistance, Bethe's carbon cycle in the theory of energy production in the sun, and the phase diagram for helium.

It is regrettable that the beautiful experimental work of Osborne, Stimson, and Ginnings on the specific heat of water and the mechanical equivalent of heat, performed at the U. S. Bureau of Standards, was overlooked.

It has been the opinion of the reviewer that the weakest part of the first edition was the chapter on thermodynamics. Although this chapter has been somewhat improved, there still remains the fundamental objection that too many topics are treated in too sketchy a manner. Just a few topics treated carefully would have been better. It is also to be regretted that the symbolism used throughout the book was not made to conform to the suggestions of the various committees that have published lists of symbols in the last few years. Thus, (Y.M.), (S.H.), and (M.W.) are hardly to be recommended as symbols for Young's modulus, specific heat, and molecular weight, respectively. It would also be preferable to use the expression "change of phase" instead of "change of state," and to drop the expression "vapor tension" entirely.

In spite of these few objections, which are mainly pedagogic in character, Professor Cork's book remains a valuable text on the experimental aspects of heat.

MARK W. ZEMANSKY
College of the City of New York

Elastic Energy Theory

BY J. A. VAN DEN BROEK. Second edition. Pp. 298 +xvii, Figs. 95, 16×23½ cm. John Wiley and Sons, Inc., New York, 1942. Price $4.50.

Redundant or statically indeterminate structures have in recent years grown in importance both in structural and aeronautical engineering. This importance can in a measure be judged from the variety of theoretical and experimental methods which have been or are being developed to solve these problems. To mention but a few, we have in the theoretical field alone the newer relaxation methods developed by Cross and Southwell, and the older methods of slope deflection, area moments, least work, and elastic energy.

The book by Professor Van Den Broek is entirely devoted to an exposition of the elastic energy method with illustrations drawn from a wide field ranging from redundant frames and beams through shafts, rings, arches, springs, pipes, and columns. The author emphasizes the universal character of the elastic energy method with which point of view this reviewer is inclined to agree. It should, however, be added that general methods are not always the most efficient. Hamilton's principle may be the most general in dynamics, but one would not determine the range of an ordinary projectile by means of it. Professor Van Den Broek has strong convictions on the subject of elastic energy which the style of the book clearly reflects.

The basic idea underlying the elastic energy method is summarized by the author as follows: "We have introduced an auxiliary force. We have given expression to the elastic energy which is stored in structures because this auxiliary force is acting while the actual loading is being applied, and, on the basis of the law of conservation of energy we have equated this energy to the work done by the auxiliary force."

The second edition differs from the first in several respects. The notation has been changed. The fundamental equation previously written as

$$P\Delta = \int \frac{M_p M_q ds}{EI}$$

now appears as

$$F\Delta = \int \frac{mMds}{EI}.$$

This simplified notation brings the book into closer agreement with the notations of other books on the same subject.

New material has been added on piston rings, pipe problems, spiral springs, and columns. We note especially the revised proposition on page 67 referring to the choice of supports for the auxiliary loads which now reads: "In applying an auxiliary load F or M' to a redundant structure it is theoretically immaterial how we imagine the beam supported as long as it is stable (and as long as no supports or restraints are introduced other than those originally there). However, it is always permissible and generally advantageous to remove sufficient restraints from the structure to make it statically determinate before the auxiliary load F or couple M' is applied." The clause in parentheses has been added in the revision. This is a significant correction. Greater clarity might perhaps have been obtained by stating specifically that only two types of supports are permissible for the auxiliary loads: one type consisting of statically determinate rigid simple supports, the other the same type of supports as used for the actual loads.

In discussing the limitations of the elastic energy method the author should perhaps have mentioned that the elastic energy method, like the slope deflection and related methods, which are all based on the ordinary flexure formula, does not give exact bending or shear stresses at a section, but gives only total bending moments and shears. Furthermore, even the total bending moments and shears have no greater validity than the flexure formula used in the derivation of the strain energy in bending.

This volume is a comprehensive exposition of the elastic energy method and has been used as a text in courses taught by the author to seniors and advanced engineering students at the University of Michigan for over ten years. It represents a book based on the accumulated experience of teaching as well as many years of revision. The reviewer knows of no better text for a course in elastic energy. It is a book that will prove of value to all structural engineers.

M. M. Frocht
Carnegie Institute of Technology

Poisson's Exponential Binomial Limit. Table I—Individual Terms; Table II—Cumulated Terms

By E. C. Molina. Pp. 98, 20½×28 cm. D. Van Nostrand Company, Inc., New York, 1942. Price $2.75.

This statistical table by a "switching-theory" engineer of an industrial research laboratory will be useful to the expert who is already using the Poisson distribution in sampling problems and, it is to be hoped, will also help to acquaint the non-expert with the wide applicability of the Poisson distribution theory to industrial problems. From the latter point of view the two-page introduction by Molina and the following one-page discussion by H. F. Dodge on the "Application to inspection problems," excellent though they are, might have been n-ply amplified without detracting from the value of the book. The references to treatments of the subject in standard textbooks on probability can hardly satisfy the reader whose curiosity is merely aroused by a short glimpse into the vast experience of these engineers accumulated over a period of forty years.

It need hardly be emphasized that the tables are especially welcome in the present emergency when quality control in production is of paramount importance. The practice of publication of tables of proven utility by an industrial research laboratory is to be fully endorsed by all applied mathematicians. This is the second mathematical table published by the Bell Telephone System, the first having been the table of Fourier Transforms by Campbell and Foster. It is to be hoped that the interval of over a decade between the appearance of the two tables is only accidental and that more of them by this and other laboratories will be forthcoming soon.

Chaim L. Pekeris
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Seven-Place Values of Trigonometric Functions for Every Thousandth of a Degree

Compiled by Dr. J. Peters. Published and distributed in the public interest by authority of the Alien Property Custodian under License No. A-1. Pp. 370, 18½×24½ cm. D. Van Nostrand Company, Inc., New York, 1942. Price $7.50.

The tables of Dr. Peters, which were first published in Berlin in 1918, consist of two parts. In the first are given seven-place values of the sine and cosine of angles from 0° to 90° for every thousandth of a degree. In the second are given seven-place values of the tangent and cotangent, also for every thousandth of a degree. Small tables of proportional parts for purposes of interpolation are given in the first part and in the early sections of the second part. The interval of one-thousandth of a degree is more than 5 times smaller than the interval of 10^{-4} radian in the WPA tables of sines and cosines. However, this advantage is often lost by the necessity of an added computational step in converting angular measure to radians. The table of tangents and cotangents in the second part is more extensive than any other available tables (Comrie, Hayashi), and it is this second part which will be the chief attraction of the volume. The reproduction is good and so is the binding.

The Van Nostrand Company is to be commended for having brought out the table at this time. Since the outbreak of the war there have been no copies of Peters' tables for sale in this country, and in many libraries the tables have had to be put on reserve—a procedure which, in the present use of mathematical tables as adjuncts to calculating machines, means complete elimination from circulation.

It is to be hoped that publishers will follow this example and take advantage of the Alien Property Custody Act to publish sorely needed books. The two volumes of *Die Differential- und Integralgleichungen der Mechanik und Physik* by Ph. Frank and R. v. Mises should be a worth while venture, especially since both editors are now in this country.

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Division of War Research

Statistical Mathematics

BY A. C. AITKEN. Second edition. Pp. 153+vii, 13×19 cm. Interscience Publishers, Inc., New York, 1942. Price $1.65.

This volume from the series of small University Mathematical Texts is destined to rank as a classic. In the opening chapter the author distills, into the remarkably short space of ten pages, the literature on the history and philosophy of statistical and probability theory. Thereafter he runs through the entire gamut of probability principles and statistical techniques: probability and frequency distributions, practical curve fitting with standard curves, probability and frequency in two variates, the method of least squares, multivariate correlation, polynomial and harmonic regression, probability distributions of statistical coefficients and, in addition, many associated subjects which do not appear in the chapter headings. The exposition carries the reader immediately to the kernel of each subject dealt with and is as clear as space limitations permit.

The book has no preface so that the audience for which it is written is itself a question of probability; one not easy to answer. The reviewer assumes that the author deliberately ignored this question and concentrated on the task of compressing as much as possible of his extensive and profound knowledge into the small space allotted to him by the general plan and scope of these University Mathematical Texts.

Aitken's *Statistical Mathematics* offers a splendid résumé to one who has digested the contents of at least half a dozen standard works on statistics, or probability; it will coordinate his previous reading and greatly improve his general perspective of the subject. The engineer, or physicist, will find the going hard. But it will pay him to master the text from *a* to *z* even if such mastery calls for considerable collateral reading. The statements in this paragraph are colored by comments from colleagues to whom the reviewer presented copies of the book under consideration.

E. C. MOLINA
Office of Scientific Research and Development, New York City

From comments by A. Nadai in his review of *Theory of Plates and Shells* by S. Timoshenko, which appeared in the *Applied Mechanics Journal* of the American Society of Mechanical Engineers, the following notes are taken:

This book is concerned with the distortion of elastic bodies in which one dimension is small compared with other dimensions, i.e., the bending theory of thin plates and the deformation of curved shells. The first chapter entitled "The bending of long rectangular plates to a cylindrical surface" consists essentially of the elementary bending theory of slender bars under a lateral and axial load. The second chapter, "Pure bending," contains formulae for the slopes, curvatures, and bending moments of uniformly bent plates for different directions, while the third chapter discusses the bending of circular plates having a rotational symmetric distortion.

The fourth, entitled "Small deflections of laterally loaded plates," includes a discussion of the boundary conditions for plates. The four boundary conditions considered are: the built-in, the simply supported, the free edge, and the elastically supported edge. Using rectangular coordinates x, y the simply supported edge is defined as follows: "If the edge $y=0$ of the plate is simply supported the deflection w along this edge must be zero. At the same time this edge can rotate freely with respect to the axis, i.e., there are no bending moments M_y along this edge." Dr. Nadai in his review objects to the assumption that he believes is unwarranted that along a simply supported edge the deflection of the plate "must be zero." The experiments of C. Bach and of M. Bergstraesser have shown that a rectangular plate carrying transversal forces (weight) can rest freely on a smooth rigid edge situated in a plane but the deflections will be zero only in certain points along the edge. A student of plate theory who uses these boundary conditions must remember that they are of a mathematical nature and should satisfy himself with a further investigation of what happens also to the transversal shearing forces along the edges of the plate.

After the introductory chapters a large number of special cases are reported in detail, such as rectangular plates with various end conditions, plates of various shapes, stretched and bent plates and those with large deflections. The last three chapters of the book deal with the deformation of shells without and with bending, including the theory of cylindrical and spherical shells.

Contributed Original Research

Flow and Recovery Properties of Viscose Rayon Yarn*

JACK J. PRESS**
Polytechnic Institute of Brooklyn, Brooklyn, New York
(Received December 21, 1942)

A series of flow and recovery tests are used as a basis for describing and interpreting the elastic, viscous, and tensile properties of a typical solid linear high polymer. Extensions of viscose rayon at 70°F and 65 percent R.H. are given under loads of 0.1 to 1.0 grams per denier (1900 to 19,000 lb. per sq. inch) for times up to 10^6 seconds. Recoveries at 70°F are given for times up to 10^6 seconds at 65 percent R.H. and also after wetting and reconditioning to remove internal strains. The data are interpreted in terms of changes in modulus, viscosity, restoring forces, viscous flow, crystallization, and internal load distribution.

INTRODUCTION

A SURVEY of literature on solid linear high polymers reveals considerable information on flow (1–20)† and recovery (21–24)† properties in the test range of seconds, hours, and days; however, there is very little information (25–29)† on the systematic use of combined flow and recovery tests. This paper indicates how such a combination of tests can contribute to a more complete description and understanding of the elastic, viscous, and tensile properties (30–90)† of such materials.

Cellulose, rubber, nylon, and wool, typical linear high-polymers, are remarkable for their unique combination of toughness, long-range elasticity, and high strength. They can conveniently be pictured as having originated in a solution or melt of long chain randomly distributed, kinked macromolecules with a comparatively low average degree of molecular interaction. Upon cooling, coagulation, or drying, these molecules are brought into closer contact, resulting in a random increase in the degree of molecular interaction. As a result of this process, the internal viscosity increases so rapidly that, on the whole, the segments or links of the long chain molecules cannot orient and rotate into suitable positions for strong interaction (crystallization). Thus we arrive at a loose, tangled, net-like structure of disordered long chain molecules interlocking at the few points where the segments happen to be suitably disposed for strong interaction. The regions of strong interaction (relatively crystalline) contribute mostly to high modulus and strength. The regions of weak interaction (relatively amorphous) contribute to long-range elasticity and the ability to distribute stresses.

Upon the application of an external force the regions with varying degrees of order approach new equilibrium positions of higher order by slippage, stretching, orientation, and rotation. The time-extension test under constant load is the simplest type of experiment with which to study such phenomena.

A sample of commercial viscose rayon (150 denier multifilament yarn) was used in the following experiments as representative of the more complicated type of composite high-polymeric system. The conditions of 70°F and 65 percent R. H. (relative humidity) were used for two reasons. (1) They yield internal viscosities great enough satisfactorily to spread the superimposed relaxation and retardation times characterizing the regions of varying degrees of order and size. (2) These conditions are fairly representative of

* This is a portion of a thesis submitted in partial fulfillment of requirements for the degree of Bachelor of Science in Chemistry at the Polytechnic Institute of Brooklyn.
** Chemist, Research Department, Virginia-Carolina Chemical Corporation.
† See Bibliography.

industrial practice, and therefore possibly of some immediate value.

Table I gives the results of such a time-extension test under a moderate load of 0.67 g/d†† for the time range of 10^{-2} to 4.5×10^{5} seconds. These data are plotted in Fig. 1 as the logarithm of time *versus* the extension. The extension at 10^{-2} second represents the instantaneous elastic elongation (0.67 percent) characteristic for the elastic modulus (1.33×10^{11} dynes/cm²) of the material. From 10^{-2} to 10^{3} seconds the curve indicates a fairly complicated change in equilibrium within those regions possessing low relaxation or retardation times. A low relaxation time is synonymous with ease or viscous flow, while a low retardation time corresponds to a slightly retarded elasticity. From 10^{3} to 4.5×10^{5} seconds the results are fairly well characterized by a straight line in the semilogarithmic graph indicating a more uniform change in equilibrium in regions having much higher relaxation and retardation times.

This fairly sharp change at about 10^{3} seconds, characteristic of all loads above 0.4 g/d, indicated the advisability of studying separately the regions from 10^{-2} to 10^{3} seconds and from 10^{3} to 10^{6} seconds.

TABLE I. Extensions (%) after varying lengths of time (seconds) under a constant load of 0.67 g/d.

Seconds	%	Seconds	%
.01	0.67*	2,000	6.9
1	2.6	3,000	7.1
5	3.2	4,000	7.2
10	3.6	5,000	7.25
20	4.2	7,500	7.35
40	4.9	11,000	7.55
60	5.1	68,000	8.45
80	5.3	75,000	8.5
120	5.55	97,200	8.6
180	5.8	183,600	8.8
250	6.0	327,600	8.9
400	6.1	356,400	8.95
700	6.4	414,000	9.05
1,000	6.6	442,800	9.08
1,500	6.8		

* Calculated from elastic modulus.

EXPERIMENTAL PROCEDURES

The material tested was a 150 denier* 40 filament Bright Viscose Yarn** with very low twist. All samples were conditioned (70°F and 65 percent R.H.) at least 24 hours before testing.

Very Short Time or Modulus Tests

One end of the sample was firmly held in a fixed position while the lower end was clamped to a little carriage of suitable weight. The carriage was so guided that a thin metal needle fastened to it was kept at a constant distance from a thin paper chart backed by a metal plate. The output of an automotive hi-tension spark transformer operating at a breaker rate of 200–300 breaks per minute was imposed between the carriage and the metal plate supporting the chart. Every time the electrical circuit was closed, by means of a transmitter key in the line, the current would flow between carriage needle and chart plate at the rate of 200–300 times per minute. With the current on and the carriage dropping rapidly the holes burned in the paper were very light and small. With a 20″ test length and at loads below 0.4 gram/denier the carriage would drop rapidly to an extension characteristic of its elastic modulus; from then on, as the retarded elasticity made itself felt, the carriage would continue to drop, but at a greatly reduced rate. At the point where the rate of drop decreased, the spark burns in the chart became much darker and larger. The first one or two spark burns were always faint, indicating that the reading taken after the second or third spark corresponded to a time interval of approximately 10^{-2} second. This gave a short time static value for the elastic modulus agreeing very well (Table II) with the best values in the literature.

Flow Tests of Table I and Fig. 1

The same equipment was used as in the modulus test. However, instead of the spark being on during the duration of the test, it was imposed periodically to mark the chart after given time intervals.

Short and Long Time Extension and Recovery Tests

The sample was clamped at the top, an 18.75″ length was marked off along its length with a

†† For viscose rayon 1 gram per denier equals approximately 19,000 lb. per square inch.

* A 150 denier yarn has about 30,000 yards to the pound.

** Courtesy of Du Pont Rayon Company.

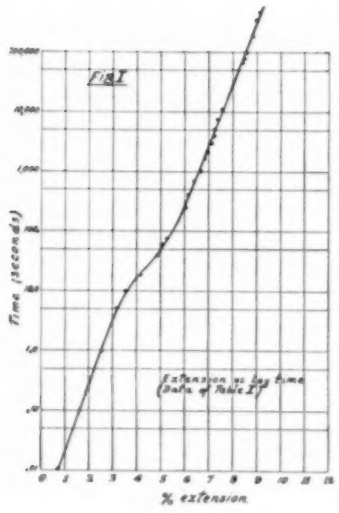

FIG. 1.

small strip of tape and then the load was applied to the lower end. The distance from the top clamp to the tape was measured periodically with a ruler to an accuracy of about 1/64th of an inch. The conditioned recovery was determined by removing the load and measuring the recovered length periodically. The wet recovery was determined by soaking the sample in water for 24 hours, conditioning for 24 hours, and then remeasuring.

RESULTS AND DISCUSSION

The short time tests consisted in measuring the extension of individual samples after 1000 seconds loading, after 1000 and 86,400 seconds recovery, and after wetting the samples and reconditioning at 70°F and 65 percent R.H. The loads ranged from 0.1–1.0 gram/denier. The results of these tests are given in Table III and Fig. 2.

An examination of Fig. 2 discloses some very interesting relationships. Up to 0.3 g/d and ¾ percent extension the time for complete recovery (1000 seconds) is the same as the time for the extension (1000 seconds). This range, although retarded, is completely elastic, involving no permanent change in the sample. It probably represents the maximum movement (mostly in the relatively amorphous regions) of various molecular segments before the weaker secondary bonds begin to break.

From 0.3 g/d and ¾ percent extension to 0.55 g/d and 4 percent extension the time for complete conditioned recovery is much longer than the time for extension. However, after wetting and reconditioning, the samples do recover completely to the same extension (−0.5 percent) as the blank after similar wetting and reconditioning. The greatly retarded conditioned recovery indicates an increase in viscosity, while the complete recovery after wetting to remove internal strains and then reconditioning indicates that no permanent deformation has taken place. The 0.5 percent shrinkage of the blank after wetting and reconditioning is due to residual internal strains in this sample of yarn.

Above 0.55 g/d and 4 percent extension the conditioned recovery is retarded and the recovery even after wetting and reconditioning is still not complete. Here again the greatly retarded recovery indicates an increase in viscosity, while the incomplete wet recovery represents permanent deformation.

Long time tests were then run in which the extensions were checked periodically under load (0.1–0.8 g/d), during recovery, and after wetting and drying. The results of these tests are shown in Table IV and are summarized in Fig. 3. We see here several features which show surprising similarity to some features of the short time tests (Fig. 2).

The composite results may be outlined as follows:

1. The recovery rate for extension up to ¾ percent is practically the same as the extension rate. Thus, we have a retarded, completely elastic range involving no permanent change in the sample and limited by a *yield extension of ¾ percent.* This probably represents the maximum

TABLE II. Comparative elasticity moduli of viscose rayon yarn.

Condition	Method	Modulus in dynes per sq. cm $\times 10^{10}$	
Air dried	Supersonics	11.5	(74)*
−190°	Static, extrapolated	13.2	(74)*
70°F and 65% R.H.	Short time static	13.3	This paper
Air dried	Normal static	8.3	(74)*
———	Normal static	5.9	(74)*
———	Normal static	6.9	(83)*
50% R.H.	Normal static	6.9	(18)*

* References at end of article.

movement of molecular segments before the weaker secondary bonds in the relatively amorphous regions begin to break.

2. Between $\frac{3}{4}$ percent and 4 percent extension, we have an elastic region in which the conditioned recovery is greatly retarded; however, after wetting to remove internal strain and then reconditioning, the recovery is complete. The extension is the important factor in this range. The incomplete conditioned recovery indicates an increase in recovery viscosity or a decrease in the restoring forces while the complete recovery after wetting to remove internal strain and then reconditioning indicates that no permanent deformation has taken place. In the relatively amorphous regions some secondary bonds probably have been broken and some segments have begun to change positions, but do not have sufficient translational and rotational freedom to move into new equilibrium positions of strong interaction. There are still strong internal restoring forces due to molecular entanglement and cross bonding, however, they are now not strong enough to overcome the steric hindrance produced by the movement of some segments to new positions. Upon wetting, water penetrates the amorphous regions, pushes apart the molecular segments and reduces the steric hindrance so that now the weakened restoring forces can bring the segments back to their original state of lower order. The 0.5 percent shrinkage of the blank after wetting and reconditioning indicates that the sample was dried during production or was wound afterwards under a tension which counteracted part of the internal restoring forces and left the material with some residual strain.

TABLE III. Extension (short time) under load for 1000 seconds, after recovery for 1000 and 86,400 seconds, and after wetting and drying. Each value is an average of five tests.

Load (grams/denier)	Extension		Recovery length	
	1000 seconds	1000 seconds	86,400 seconds	Wet and dried
Blank				−.50%
.2	.50%	.01%		
.3	.71	.02	−.02%	−.46%
.4	1.19	.19	.02	−.42
.5	2.98	1.09	.58	−.45
.65	5.66	3.13	2.39	−.16
.8	8.64	5.20	4.30	.25
1.0	10.97	6.96	6.18	.85

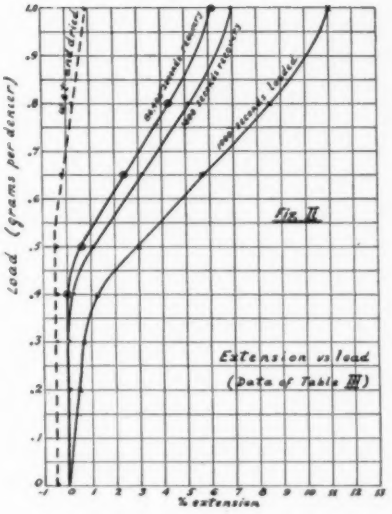

FIG. 2.

3. Above 4 percent extension the conditioned recovery is greatly retarded and the sample *does not* recover completely even after wetting and reconditioning. There is, in this region, an increase in recovery viscosity or a decrease in restoring forces with some molecular segments now having sufficient energy to move into new equilibrium positions. The extension of 4 percent appears to be associated with a *critical energy level* in the amorphous regions, which is sufficiently great to allow some segments to move into new equilibrium positions.

4. The logarithmic rate of extension (Fig. 4) for each load from 10^3 *to* 10^6 seconds can be fairly well represented by a straight line described by the equation $\gamma = A + b \log (t+1)$. Here γ is the extension at time t, A is a constant, and b is the slope of the line. A plot of slope b against load in grams/denier (Fig. 5) shows that the logarithmic rate of extension increases proportional to load up to 0.2 g/d, very rapidly for loads from 0.2 to 0.4 gram/denier, and then remains fairly constant for the loads above 0.4 g/d. Referring these load ranges back to Fig. 4 we see that each one corresponds exactly to one of the ranges of extension which characterize the material. The slopes (b) for loads up to 0.2 g/d, proportional to load, have reached a maximum extension of $\frac{3}{4}$ percent for the 0.2 g/d load at 10^6 seconds. This range, therefore, is within the $\frac{3}{4}$ percent limiting extension (see paragraph 1) of retarded, com-

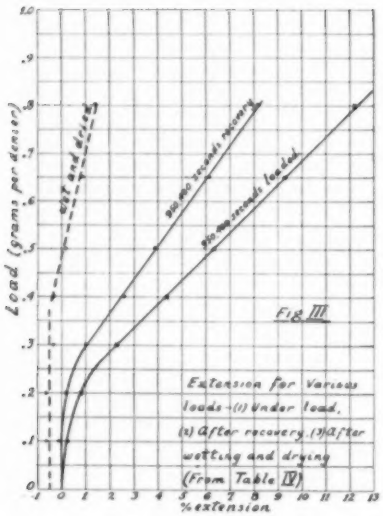
Load (grams per denier)
% extension
Fig. III
Extension for Various loads - (1) Under load, (2) After recovery (3) After wetting and drying
(From Table IV)

FIG. 3.

pletely elastic behavior and involves no permanent changes in the material. The slopes (b) for loads of 0.2 g/d to 0.4 g/d, increasing rapidly with load, cover roughly the range from $\frac{3}{4}$ percent to 4 percent extension. This is the range (see paragraph 2) previously associated with incomplete conditioned recovery and complete wet recovery, with the incomplete conditioned recovery being ascribed either to an increase in recovery viscosity with extension or to a decrease in the restoring forces. The rapid increase in slope (Fig. 5) for this range represents a sharp drop in viscosity, indicating that the incomplete conditioned recovery is due mostly to a decrease in the internal restoring forces resulting from the breaking of many secondary bonds in the amorphous regions. The slopes (b) for the loads above 0.4 g/d, remaining fairly constant, cover roughly the range of extension above 4 percent. This was the range (see paragraph 3) associated with incomplete conditioned recovery and incomplete wet recovery, with the incomplete conditioned recovery being ascribed either to an increase in viscosity or to a decrease in restoring forces. The fairly constant slope (Fig. 5), for this range, indicates an increase in viscosity proportional to both load and extension. From this it appears likely that the incomplete conditioned recovery for this range is due not as much to a decrease in the original restoring forces as it is to an increase in viscosity resulting from segments in the relative amorphous regions moving into new equilibrium positions of higher order.

The short time (10^3 seconds) and long time (10^6 seconds) data were then rearranged (Fig. 6) in an effort to determine whether the permanent deformation and increase in viscosity for extension above 4 percent was due to translational movement (viscous flow) or to rotational movement (crystallization) of segments. Upon calculation, the percent deformation for extensions *above 4 percent* remained fairly constant in the short time (10^3 seconds) tests at about 19 percent of the extension (0.65, 0.80, and 1.0 g/d gave values of 19.3 percent, 17.8 percent, and 19.4 percent), while in the long time (10^6 seconds) tests the percent increase in permanent deformation based on the increase in extension from 10^3 to 10^6 seconds showed an increase in percent permanent deformation with load (0.50, 0.65, and 0.80 g/d gave values of 19.2 percent, 27.2 percent, and 31.2 percent). The constancy of the percent permanent deformation for the short time (10^3 seconds) tests indicates that the mechanism is probably one of translational movement which is proportional to the extension. This represents mostly the disentangling of parts of molecules in the amorphous regions and the sliding of some segments over other segments along the fiber axis, producing an increased steric hindrance to recovery. The increase of the per-

TABLE IV. Extension (long time) under load, during recovery and after wetting and drying.

Time (seconds)	Load (grams/denier) .1	.2	.3	.4	.5	.65	.8
(loaded)	Extension (%)						
100	.0 %	.28%	.5 %	.73%	1.33%	4.61%	7.73%
1,000	.0	.28	.67	1.00	3.67	6.06	9.06
10,000	.06	.33	.89	2.33	4.50	7.62	10.16
20,000	.06	.33	1.27	2.83	4.83	7.95	10.33
86,400	.12	.62	1.62	3.50	5.50	8.50	11.00
108,000	.12	.67	1.67	3.67	5.57	8.62	11.00
194,400	.17	.67	1.83	3.83	5.83	8.83	11.33
345,600	.17	.67	2.00	4.06	6.00	9.00	11.62
540,000	.17	.73	2.16	4.33	6.16	9.16	11.84
691,200	.23	.83	2.22	4.33	6.33	9.28	12.00
864,000	.28	.83	2.33	4.39	6.33	9.33	12.17
950,400	.28	.83	2.33	4.45	6.39	9.33	12.33
(load removed)	Recovery length (%)						
100	.0	.5	1.78	3.67	5.39	8.00	10.06
1,000	.0	.495	1.67	3.56	5.16	7.51	9.62
10,000	.0	.33	1.50	3.28	4.73	7.00	9.00
20,000	.0	.33	1.45	3.16	4.67	6.67	8.73
259,200	.0	.28	1.17	2.83	4.06	6.16	8.33
518,400	.0	.28	1.12	2.67	4.00	6.06	8.28
950,400	−.05	.23	1.06	2.67	3.95	6.06	8.22
(soaked and dried)	−.07	−.62	−.33	−.33	−.17	.83	1.39

cent permanent deformation with load in the long time (10^6 seconds) tests indicates that here the increase of rotational freedom, with increasing energy stored in the molecular segments, is beginning to play an important part. The percent permanent deformation (19.2 percent) for the smallest load (0.5 g/d) is similar to the fairly constant value (approximately 19 percent) for the short time tests and is probably mostly translational movement (viscous flow). Above 0.5 g/d the percent permanent deformation in the long times tests (10^6 seconds) increases with load (27.2 percent at 0.65 g/d and 31.2 percent at 0.8 g/d) suggesting that rotational movement (crystallization) is occurring simultaneous with translational movement (viscous flow). The higher load stores more energy in the sample and increases the rotational energy level of the segments. For rotational movement the increase in recovery viscosity results mostly from the establishment of new secondary intermolecular forces which oppose the recovery forces.

The greater rotational freedom of the long time (10^6 seconds) tests as compared to the short time (10^3 seconds) tests suggests the possibility of a shift in the internal load distribution with time either from regions of strong interaction (relatively crystalline) to regions of weaker interaction (relatively amorphous) or in the amorphous regions to fewer segments as the reinforcement between segments relaxes and the load distribution becomes less regular.

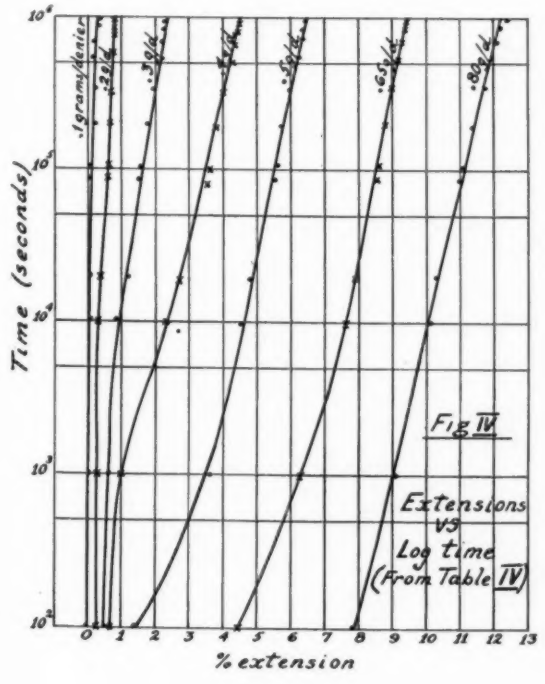

FIG. 4.

An attempt to check the decrease in viscosity and the change in load distribution, in going from $\frac{3}{4}$ percent to 4 percent extension, was made by calculating the very short time (10^{-2} second) elastic modulus at $\frac{3}{4}$ percent and 4 percent extension. The loads required to produce extensions of $\frac{3}{4}$ percent and 4 percent, respectively, after 10^2, 10^3, 10^4, 10^5, and 10^6 seconds were approximated from the curves of Fig. 7 and plotted in Fig. 8 as load (grams/denier) against log time (seconds). The points for both $\frac{3}{4}$ percent and 4 percent extension are fairly well respresented by straight lines described by an equation of the type, $S=A-b\log(t+1)$. S is the load in grams/denier required to produce the extension ($\frac{3}{4}$ percent or 4 percent) in time t, A is a constant, and b is the slope of the line.

$\frac{3}{4}$ Percent Extension

$$b=\frac{2.35}{4}=.59$$

$@t=10^3$; $\quad 3.5\text{ g/d}=A-.59\log(10^3+1)$

$$A=5.27$$

$$S=5.27-.59\log(t+1)$$

$@t=10^{-2}$; $\quad S=5.27-.59\log(10^{-2}+1)$

$$=5.27\text{ g/d}$$

$$\therefore\quad E_{3/4\%}=\frac{5.27\times1.33\times10^{9*}}{.75\%}$$

$$=9.37\times10^{11}\text{ dynes/cm}^2.$$

4 Percent Extension

$$b=\frac{2.25}{4}=.56$$

$@t=10^3$; $\quad 5.5\text{ g/d}=A-.56\log(10^3+1)$

$$A=7.18$$

$$S=7.18-.56\log(t+1)$$

$@t=10^{-2}$; $\quad S=7.18-.56\log(10^{-2}+1)$

$$=7.18$$

$$\therefore\quad E_{4\%}=\frac{7.18\text{ g/d}\times1.33\times10^{9*}}{4\%}$$

$$=2.39\times10^{11}\text{ dynes/cm}^2.$$

* 1 gram/denier $=1.33\times10^9$ dynes per cm².

Comparing the determined (small loads: t approximately 10^{-2}) short time static modulus with the calculated moduli ($t=10^{-2}$) we get:

Determined modulus

$$E=1.33\times10^{11}\text{ dynes/cm}^2.$$

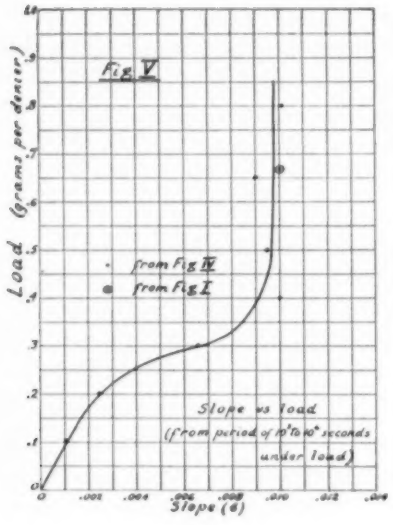

FIG. 5.

Calculated moduli

$$E_{3/4\%} = 9.37 \times 10^{11} \text{ dynes/cm}^2,$$
$$E_{4\%} = 2.39 \times 10^{11} \text{ dynes/cm}^2.$$

The modulus apparently increases with extension up to a maximum of about 10^{12} dynes/cm² at $\frac{3}{4}$ percent extension, decreases to about 2.5×10^{11} dynes/cm² at 4 percent extension, and then probably starts to increase again. The modulus increases with extension up to $\frac{3}{4}$ percent as the load is distributed amongst an increasing number of regions and segments, from $\frac{3}{4}$ percent to 4 percent extension the modulus and viscosity decrease as more and more of the weaker secondary bonds in the relatively amorphous regions break, and then above 4 percent extension the modulus and viscosity increase as reinforcement takes place in the relatively amorphous regions. Several other observations on the relation of time to tenacity have some bearing on this problem. The short time tests (Fig. 6) show a slightly stronger reinforcement for the 1.0 g/d test as compared with the 0.80 g/d test. The long time 1.0 g/d tests were impossible to complete, all samples breaking before 10^6 seconds. This indicates a strong short time reinforcement which gradually relaxes until the sample breaks. Possibly the short time reinforcement of segments in the amorphous regions allows these regions at first to support a high load, then gradually the reinforcement relaxes, fewer and fewer segments are in a position to support the load and finally the sample breaks. From this we may obtain a value of 0.8–1.0 g/d for the low limiting tenacity of the material with minimum reinforcement. The normally determined tenacity for this material is about 2.0 g/d conditioned and about 1.0 g/d wet. The long time tenacity of 0.8–1.0 g/d checks fairly well with the normal wet tenacity of about 1.0 g/d and indicates a possible similarity in internal load distribution and breaking mechanism. In the wet tensile test, because of the presence of water molecules in the amorphous regions, the material is not able to reinforce itself significantly. Further work is in progress on the tenacity aspect of the problem.

SUMMARY

1. The relation of load and time to the elastic properties of viscose rayon varies with the range of extension and up to at least 12 percent may be roughly divided into three ranges; to $\frac{3}{4}$ percent, from $\frac{3}{4}$ percent to 4 percent, and above 4 percent.

2. Extensions to a $\frac{3}{4}$ *percent yield extension* are retarded and completely elastic. The elastic modulus increases from 1.33×10^{11} (deter-

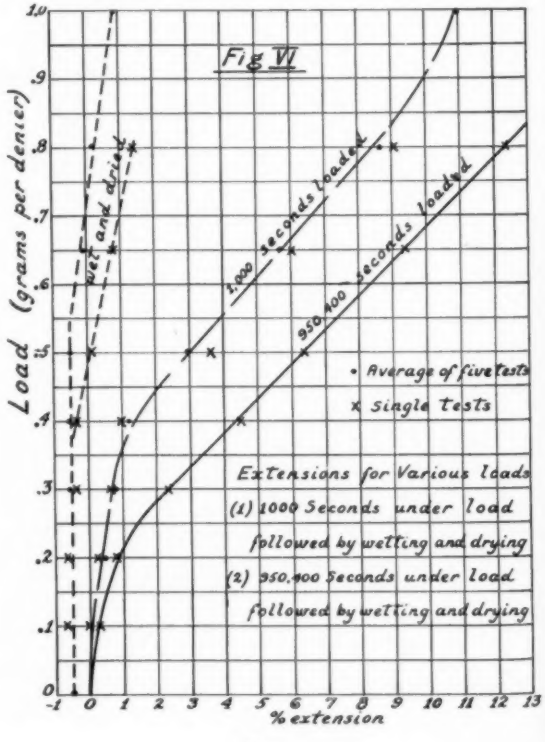

FIG. 6.

mined) to 9.37×10^{11} dynes/cm² (calculated). This range represents the maximum movement of molecular segments before weaker secondary bonds in the relatively amorphous regions begin to break.

3. The second range from $\frac{3}{4}$ percent to 4 percent extension is not completely recoverable until the sample is swollen with water. The elastic modulus decreases from 9.37×10^{11} dynes/cm² (calculated) at $\frac{3}{4}$ percent extension to 2.39×10^{11} dynes/cm² (calculated) at 4 percent extension. Numerous secondary bonds in the relatively amorphous regions are broken resulting in a decrease in restoring forces and viscosity. Molecular chain segments have begun to change positions, but do not have sufficient translational and rotational energy to move into new equilibrium positions of strong interaction. Swelling in water reduces the steric hindrance and allows the weakened restoring forces to bring the segments back to their original state of lower order.

4. Above 4 percent the extensions are not completely recoverable even after swelling in water. The further decrease in viscosity with breakage of additional secondary bonds in the relatively amorphous regions is overshadowed by the increase in viscosity resulting from some molecular segments now having sufficient translational (viscous flow) and rotational (crystallization) energy to move into new equilibrium positions of higher order. For short times (10^3 seconds) the translational movement predominates and the permanent deformation is proportional to the extension above a critical value of about 4 percent. For longer times (10^3 to 10^6 seconds) the rotational movement, which is proportional to load, becomes apparent and is superimposed over the translational movement.

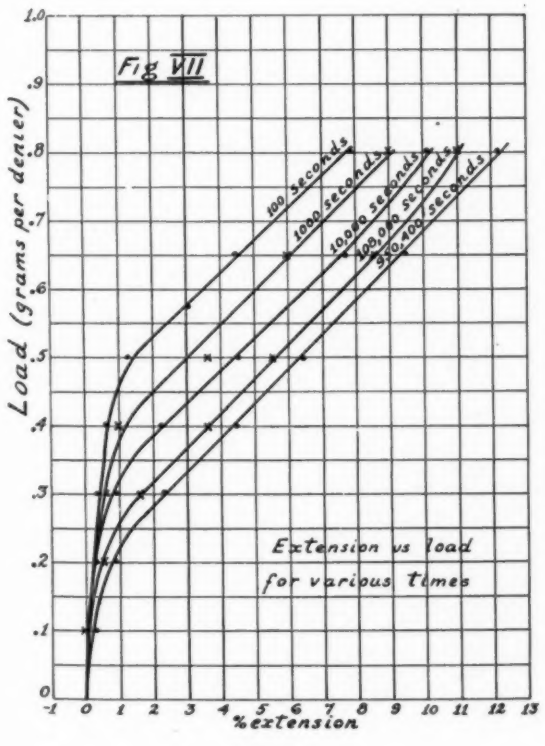

FIG. 7.

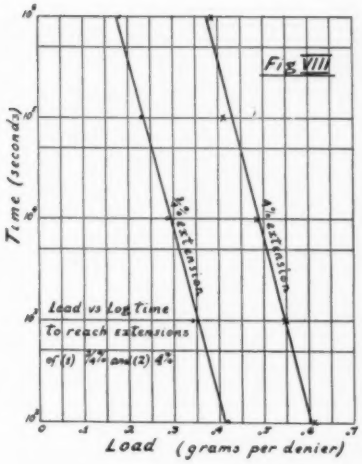

FIG. 8.

5. With loads of 1.0 grams/denier (approximately 19,000 lb. per sq. inch) this sample of viscose rayon shows strong reinforcement in the short time (10^3 seconds) tests and breaks in the long time (10^3 to 10^6 seconds) tests. The short time reinforcement in the amorphous regions allows them at first to support a high load, then gradually the reinforcement relaxes until a low limiting tenacity of 0.8–1.0 gram/denier is reached for the material with minimum reinforcement. The normally determined tenacity is about 2.0 grams/denier dry and about 1.0 g/d wet. The similarity of the long time tenacity with the normal wet tenacity indicates a possible similarity in internal load distribution and breaking mechanism.

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BIBLIOGRAPHY

Flow

(1) J. Delmonte and W. Dewar, "Factors influencing creep and cold flow of plastics," Silk and Rayon (Nov. 1941), p. 690.
(2) J. E. Field, "An x-ray study of the proportion of crystalline and amorphous components in stretched rubber," J. App. Phys. **12**, 23 (1941).
(3) E. Freund and F. Deutsch, "Spontaneous extension of cellulose acetate films," Rayon Tex. Monthly (May, 1940).
(4) M. Harris, unpublished results.
(5) H. Hintenberger and W. Neumann, "The behavior of raw rubber when stretched isothermally," Rubber Chem. and Tech. **11**, 647 (1938).
(6) H. Hintenberger and W. Neumann, "Stress-strain and strength anomalties of (latex) rubber," Naturwiss. **1**, 13 (1938).
(7) I. Karger and E. Schmid, Zeits. f. tech. Physik **6**, 124 (1923).
(8) S. S. Kistler, "The thermoplastic behavior of linear and three-dimensional polymers," J. App. Phys. **11**, 769 (1940).
(9) K. H. Meyer, "Researches on rubber and rubber-like substances," Chem. and Ind. **57**, 439 (1938).
(10) F. T. Peirce, Shirley Inst. Mem. **2**, 278 (1923).
(11) L. E. Peterson, R. L. Anthony, and E. Guth, "Stress-temperature relations and creep for synthetic rubbers," Am. Chem. Soc. Meeting, Buffalo, New York (Sept. 1942).
(12) Shearman, Sci. J. Roy. Coll. Sci. **11**, 59 (1941).
(13) S. E. Sheppard and J. G. McNally, "The rate of deformation of cellulose nitrate-camphor films under static stresses," J. Phys. Chem. **35**, 2498 (1931).
(14) S. E. Sheppard and J. G. McNally, "Thermoelastic effect in cellulose ester films," J. Phys. Chem. **35**, 100 (1931).
(15) S. E. Sheppard, E. K. Carver and S. S. Sweet, "Time factor and yield value of cellulose esters," Ind. Eng. Chem. **18**, 76 (1926).
(16) J. B. Speakman and S. Y. Shah, "The plasticity of animal fibers," Soc. Dyers and Colourists **57**, 108 (1941).
(17) J. B. Speakman, "The gel structure of the wool fiber," J. Tex. Inst. **17**, 1457 (1926).
(18) R. L. Steinberger, "Creep in cellulose acetate filaments, creep in cuprammonium filaments, creep in single fibers of American delta cotton," Tex. Res. **6** (1936).
(19) L. R. G. Treloar, "Crystallisation phenomena in raw rubber," Trans. Faraday Soc. **37**, 84 (1941).
(20) H. J. Woods, "X-ray studies of the structure of hair, wool, and related fibers. IV. The molecular structure and elastic properties of the biological cells," Roy. Soc. London **166**, 92 (1938).

Recovery

(21) G. E. Collins, Shirley Inst. Mem. **1**, 133 (1922).
(22) G. E. Collins, J. Tex. Inst. **30**, 46 (1939).
(23) G. S. Whitby, Trans. Inst. Rubber Ind. **6**, 40 (1930).
(24) W. Weltzein and K. Gotze, *Chemische und physikalische Technologie der Kunstseiden* (Leipzig, 1930), p. 125.

Flow and Recovery

(25) O. Eisenhut, "New methods of testing staple fiber," Mell. Textilber. No. 9, p. 625 (1939).
(26) R. N. Haward, "The extension and rupture of cellulose acetate and celluloid," Trans. Faraday Soc. **38**, 394 (1942).
(27) H. Leaderman, "Textile materials and the time factor. I. Mechanical behavior of textile fibers and plastics," Tex. Res. **11**, 171 (1941).
(28) L. R G. Treloar, "Elastic recovery and plastic flow in raw rubber," Trans. Faraday Soc. **36**, 538 (1940).
(29) L. R. G. Treloar and G. Gee, "Relation between molecular weights and physical properties of rubber fractions," Rubber Chem. and Tech. **14**, 580 (1941).

General

(elastic, viscous, tensile properties; structure)

(30) J. Allen, "The tenacity of cellulose acetate in various forms," Rayon and Melliand Textile Monthly **57**, 233 (1936).
(31) T. Barrath, "Elastic modulus of cellulose fibers," J. Text. Inst. **13**, 22 (1922).
(32) S. L. Bass, T. A. Kauppi, and J. L. Sherk, "Evaluation of properties of ethyl cellulose by the use of load-elongation curves (II Resins)," Paint, Oil, and Chem. Rev. (May 13, 1937).
(33) S. L. Bass and T. A. Kauppi, "Evaluation of ethylcellulose by load-elongation curves," Ind. and Eng. Chem. **29**, 678 (1937).
(34) K. Bennewitz and H. Roetger, "Elasticity and plasticity of high polymers," Physik. Zeits. **40**, 416 (1939).
(35) E. E. Berkley, "Cellulose orientation, strength, and cell wall development of cotton fibers," Tex. Res. **9**, 355 (1939).
(36) J. H. de Boer, "Strength of cellulose and molecular structure," Trans. Faraday Soc. **32**, 10 (1935).
(37) N. A. de Bruyne, "Note on viscous and plastic flow," Proc. Phys. Soc. **53**, 251 (1941).
(38) J. M. Burgers, *First Report on Viscosity and Plasticity* (New York, 1939), second edition; *Second Report on Viscosity and Plasticity* (Amsterdam, 1938).
(39) W. N. Findley, "Mechanical tests of cellulose acetate," Trans. A. S. T. M. (June, 1941).
(40) C. S. Fuller, W. O. Baker and N. R. Pape, "Crystalline behavior of linear polyamides. Effect of heat treatment," J. Am. Chem. Soc. **62**, 3275 (1940).
(41) C. S. Fuller, W. O. Baker, and N. R. Pape, "Effects of heat, solvents, and hydrogen-bonding agents on the crystallinity of cellulose esters," J. Am. Chem. Soc. **64**, 776 (1942).
(42) C. S. Fuller and W. O. Baker, "Macromolecular disorder in linear polyamides. Relation of structure to physical properties of copolyamides," J. Am. Chem. Soc. **64**, 2399 (1942).
(43) B. S. Garvey, Jr., C. H. Alexander, F. E. Kung and D. E. Henderson, "Mixed polymers and vulcanizable plasticizers," Ind. Eng. Chem. **33**, 1060 (1941).
(44) E. Guth and H. M. James, "Elastic and thermoelastic properties of rubber-like materials," Ind. Eng. Chem. **33**, 624 (1941).
(45) M. Harris, L. R. Mizell, and L. Fourt, "Elasticity of wool as related to its chemical structure," J. Research, Nat. Bur. Stand. **29** (1942).
(46) M. Harris and A. M. Sookne, "Stress-strain characteristics of wool as related to its chemical constitution," J. Research, Nat. Bur. Stand. **19** (1937).

(47) L. Hartshorn, N. J. L. Megson, and E. Rushton, "Molecular relaxation and the elastic and dielectric properties of plastics," Proc. Phys. Soc. **52**, 817 (1940).
(48) P. H. Hermans, "The analogy between the mechanism of deformation of cellulose and that of rubber," J. Phys. Chem. **45**, 827 (1941).
(49) K. Hohenemser and W. Frazer, "Plasticity and elasticity," J. Rheol. **3**, 16 (1932).
(50) W. L. Holt and A. T. McPherson, "Change of volume of rubber on stretching effects of time, elongation, and temperature," J. Research, Nat. Bur. Stand. **17** (1936).
(51) W. L. Holt and F. L. Roth, "Tensile properties of rubber compounds at high rates of stretch," J. Research, Nat. Bur. Stand. **23** (1939).
(52) W. Holzmoeller and E. Jenckel, "Elastic behavior of high polymers," Zeits. f. physik. Chemie **A186**, 359 (1940).
(53) R. Houwink, "The structure of rubber," Kolloid Zeits. **95**, 303 (1941).
(54) R. Houwink, *Elasticity, Plasticity, and Structure of Matter* (Oxford, 1937).
(55) J. Karger and E. Schmid, "Elastic modulus of native cellulose," Zeits. f. tech. Physik **6**, 124 (1925).
(56) O. Kratky, "The micellar structure of fiber substances," Silk and Rayon (August, 1937).
(57) O. Kratky, "The network structure of cellulose," Silk and Rayon (June, 1939).
(58) W. Kuhn, "The relation between molecular size, statistical molecular shape and elastic properties of highly polymerized substances," Kolloid Zeits. **76**, 258 (1936).
(59) W. Kuhn, "Relations between viscosity and elastic properties of amorphous substances," Zeits. f. physik. Chemie **B42**, 1 (1939).
(60) H. Leaderman, "Plastics and the time factor," Am. Dyestuff Rep. **31**, 227 (1942).
(61) E. Mack, Jr., "The structure of rubber and the mechanism of elastic stretching," J. Am. Chem. Soc. **56**, 2757 (1934).
(62) H. Mark, "Fundamental paper on elasticity and structure of rubber," J. Phys. Chem. **41**, 221 (1937).
(63) H. Mark, "Strength of cellulose as derived from molecular structure," Melliand Text. **10**, 695 (1929).
(64) H. Mark, "Intermicellar hole and tube system in fiber structure," J. Phys. Chem. **44**, 764 (1940).
(65) H. Mark and T. Alfrey, "A statistical treatment of crystallization phenomena in high polymers," J. Phys. Chem. **46**, 112 (1942).
(66) H. Mark, "Structure and mechanical behavior of high polymers," Cold Spring Harbor Symposia on Quantitative Biology **9**, 204 (1941).
(67) H. Mark and T. Alfrey, "Phase transitions in rubber and their connection with the stress-strain curve," Rubber Chemistry and Technology **14**, 525 (1941).
(68) H. Mark, "Intermolecular forces and mechanical behavior of high polymers," Ind. Eng. Chem. **34**, 1343 (1942).
(69) H. Mark, "Relation between chain length distribution curve and tenacity," Paper Trade J. (July 17, 1941).
(70) H. Mark, "Phase transition and elastic behavior of high polymers," Ind. Eng. Chem. **34**, 449 (1942).
(71) H. Mark and E. H. Freund, "Internal structure of synthetic fibers and its influence on their physical properties," Rayon Text. Monthly **23** (1942).
(72) H. R. Mauersberger, "Elongation *versus* tension in fine denier viscose rayon yarns," Rayon and Melliand Text. Monthly (Aug. 1934).
(73) J. C. Maxwell, "Fundamental paper on relaxation and flow," Phil. Mag. [12], **35**, 134 (1868).
(74) K. H. Meyer and W. Lotmar, "Elastic modulus of cellulose preparations," Helv. Chem. Acta **19**, 68 (1936).
(75) K. H. Meyer and H. Mark, *High Polymeric Chemistry* (New York 1940 and 1941), Vols. 2 and 4 of the series on high polymers.
(76) G. Mikhailov and V. Kirilina, "Study of elastic relaxation by a resonance method," Rubber Chem. and Tech. **14**, 858 (1941).
(77) D. R. Morey, "The relation of orientation to physical properties of cottons and rayons," Tex. Res. **5**, 483 (1935).
(78) F. H. Muller, "Molecular states of order and some of the resulting properties of macromolecular substances with filament and net structure," Kolloid Zeits. **95**, 306 (1941).
(79) F. H. Muller, "Molecular states of order and some of the resulting properties of macromolecular substances with filament and net structure," Kolloid Zeits. **95**, 138 (1941).
(80) F. H. Muller, "Formation and properties of fiber and molecular structure," Physik. Zeits. **42**, 123 (1941).
(81) R. F. Nickerson, "Structure, properties, and utilization of cotton," Ind. Eng. Chem. **34**, 1149 (1942).
(82) R. F. Nickerson, "Cotton fibers," Ind. Eng. Chem. **32**, 1454 (1940).
(83) B. H. Wilson, "Rayonee," Silk and Rayon (Jan. 1940), p. 30.
(84) J. R. Scott, "The plastic-elastic behavior of ebonite," Trans. Faraday Soc. **38** (1942).
(85) W. A. Sisson, "X-ray studies of crystallite orientation in cellulose fibers. III," J. Phys. Chem. **44**, 513 (1940).
(86) J. B. Speakman, "Stress-strain characteristics of wool as related to its chemical constitution," Am. Dyestuff Reptr. 168 (April, 1938).
(87) H. M. Spurlin, "Mechanical properties and distribution curve," Ind. Eng. Chem. **30**, 538 (1938).
(88) L. R. G. Treloar, "Thermodynamic study of the elastic extension of rubber," Trans. Faraday Soc. **38** (1942).
(89) W. Weltzien, "Elasticity and plasticity of cellulose," *Technologie der Kunstseide* (Leipzig, 1930), p. 33.
(90) H. J. Woods, "Effect of humidity and temperature on the Young's modulus of keratin fibers," Proc. Leeds Phil. and Lit. Soc., Sci. Sec. **3**, 577 (1940).

Studies in Lubrication

XI. Temperature Behavior of Journal Bearing Systems

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Transient bearing temperature and friction data have been obtained for a journal bearing system operating under thick film conditions. The journal speed was varied from 1.5 to 11.0 r.p.s., and tests were made with five different lubricants. Equilibrium values of temperature and friction were determined by exponential extrapolation of data taken over time intervals of three to six hours. By inversion of the Petroff formula, average film temperatures were calculated from the friction data. These were in all cases greater than the directly measured bearing temperatures, although the excess was in general less than 10°F. The equilibrium temperature data for the various lubricants and speeds were correlated on the basis of heat balance considerations by the formula:

$$\mu N^2 = \text{const}\ (T - T_a)^{1.35},$$

the film viscosity μ and temperature T referring to equilibrium conditions, T_a denoting the ambient temperature, and N the journal speed. This relation permits the calculation of the equilibrium conditions for a journal bearing system of the type used here for any lubricant and journal speed. The asymptotic rate of approach to equilibrium was also quantitatively correlated to the final equilibrium conditions. For Gulflube 10 the equilibrium frictional force was found to increase monotonically with the journal speed, the rate of increase quickly tapering off for speeds greater than 4 r.p.s. Its value at 11 r.p.s. is only 0.14 of that to be expected if there were no fall in film viscosity. For Gulflube 30 the friction force curve rises to a maximum at 3 r.p.s. and then falls slowly for greater speeds. The value at 11 r.p.s. is here only 0.04 of that for a constant film viscosity system.

INTRODUCTION

ONE of the major problems still outstanding in the subject of thick film lubrication is the nature of the temperature rise and distribution in the lubricant film. A recent review of the subject by Hersey[1] reveals that definite and quantitative information on the subject is still lacking. The direct theoretical approach to this problem as outlined by Hersey, in which a balance is set up between the frictional heat generated and the heat dissipated, involves the knowledge of heat transfer data, which are not well established. Moreover, it basically represents a formulation of the problem limited to steady state conditions.

Many observations have been made, and some have been reported in the literature, on the ultimate temperature rise in journal bearing systems after the latter have reached temperature equilibrium. However, but few systematic data are available on the relationship of such observed temperature elevations to the operating conditions of the lubrication system, such as shaft speed or lubricant characteristics.[2] Nor has the transient history of the temperature rise been studied in detail. Moreover, the temperatures, as generally observed, have been those measured with thermocouples embedded in the bearing metal at various depths from the actual bearing surface. This latter circumstance thus leaves open the question as to the relationship between the observed temperatures and those actually obtaining within the lubricant film. Of course, the same question would remain if the thermocouples had been embedded within the bearing surfaces themselves.

In the following will be reported data on the film and bearing temperatures of journal bearing systems, which throw light upon the general problem of the temperature elevation of a lubricant film. One major phase of the data consists of records of the bearing temperature, obtained by thermocouples set flush with the bearing surface, throughout the transient history of the film temperature elevation, from the beginning of rotation of the shaft until equilibrium is approached. In addition, effective, or what are considered to be true average film temperatures, have been obtained by a method recently suggested in a discussion of the paper by Hersey.[3] This method essentially consists in the calculation of the film temperature from the observed

[1] M. D. Hersey, Trans. A. S. M. E. **64**, 445–50 (1942).
[2] O. Lasche, Traction and Transmission **6**, 33–65 (1903).
[3] M. Muskat, Trans. A. S. M. E. **64**, 452 (1942).

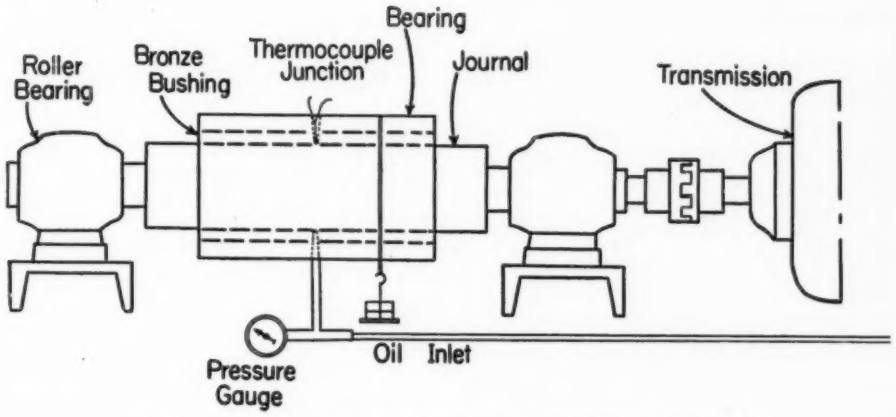

FIG. 1. Diagrammatic representation of journal bearing system.

frictional torque on the bearing. Such a calculation is based on the premise that under thick film lubrication conditions—which is believed to obtain in all of the experiments reported there—the Reynolds hydrodynamic theory is strictly valid. This basic assumption, on the other hand, has been recently verified[4,5] by direct experimental tests, not only for journal bearing systems, but also for thrust bearings, where the theoretical predictions were confirmed quantitatively by empirical observations. Acceptance of this assumption, therefore, implies that if the geometry of the bearing, the journal speed, and lubricant viscosity were known, the frictional torque or coefficient of friction could be accurately predicted. And, conversely, if the latter were measured, any one of the other factors and, in particular, the viscosity of the film, could be calculated. Once the film viscosity is determined, the film temperature is readily obtained by reference to the viscosity temperature characteristics of the lubricant. In these latter steps lies the essence of the method used here.

In applying the procedure just outlined, a journal bearing system of known geometrical characteristics was used. It was supplied by lubricants of known viscosity temperature characteristics. The frictional torques and coefficients of friction were measured, and from these were calculated the corresponding effective film viscosities. The latter gave at once the equivalent film temperatures, as already indicated.

It is to be understood that the film temperatures or viscosities as determined by the method used here represent only hydrodynamic averages over the whole lubricant film. These are, however, of primary interest from a practical point of view. On the other hand, it may be noted that in these experiments direct measurements of the temperature over the surface of the bearings showed that, at least in the condition of temperature equilibrium, the bearing temperature was quite uniform and showed but little variation over the whole extent of the surface. This, of course, does not imply that the film temperature is uniform across the thickness of the film. As theoretical calculations show, there may exist very appreciable temperature variations across the film thickness, with a maximum lying between the center of the film and the metal bounding surface of higher temperature. Nevertheless, from a practical point of view, it is felt that the average or effective film temperatures as here obtained provide a representation of the actual film conditions sufficiently accurate for all engineering purposes. In fact, even if the detailed temperature distribution could be determined and were available, the difficult problem would still remain of deriving an appropriate averaging procedure before the results could be used in engineering applications. Indeed, one of the principal advantages of the present point of view and method of study is that it automatically provides an averaging procedure of the type that is required for lubrication calculations.

[4] F. Morgan and M. Muskat, J. App. Phys. **10**, 327–34 (1939).
[5] F. Morgan, M. Muskat, and D. W. Reed, J. App. Phys. **11**, 541–48 (1940).

EXPERIMENTAL PROCEDURE

The measurements were performed with the journal bearing lubrication machine described in a previous paper[6] and shown diagrammatically in Fig. 1. The essential parts of the machine consist of a hardened steel shaft of $3\frac{1}{2}$-in. diameter, driven by a variable speed transmission, and surrounded by a bronze bushing. The original bearing was $9\frac{3}{4}$ in. long, but the effective bearing length as used in these experiments was only 5.50 in., as a result of reaming out the bearing near its ends for other experiments. The total bearing load, which was of the order of 100 lb., consisted simply of the weight of the bearing plus that of auxiliary weights used in the friction torque measurements. The latter were made by suspending appropriate weights from a string encircling the bearing in such a way as to give a resultant torque equal and opposite to that generated by the shearing stresses in the lubricant film. Series of measurements were made at journal speeds varying from approximately $1\frac{1}{2}$ to 11 r.p.s. for two oils, namely Gulflube 10 and Gulflube 30, and several special runs were also carried out on Gulflube 50, with and without 3 percent of an additive for increasing the viscosity index, and with Gulflube 10 with 10 percent viscosity index additive. The viscosity temperature characteristics of these oils are shown in Fig. 2.

Bearing temperatures were measured directly by means of thermocouples embedded in the bearing surface. The thermocouple located at the crown, or loaded portion, of the bearing was read at various intervals throughout the complete transient history of the temperature rise. After equilibrium was attained, additional readings were recorded on seven other thermocouples distributed over the whole bearing surface. Lubricant was fed to the bearing from a pressure tank through an inlet diametrically opposite the top thermocouple, and the feed pressure was maintained at approximately 12 lb. per sq. in. throughout the runs. The journal speed was determined with the aid of a counter attached to the end of the shaft, and by timing large numbers of revolutions throughout the course of the runs.

As a step preliminary to carrying out the final experiments, the applicability of the Reynolds hydrodynamic theory or, in particular, the Petroff formula, was again confirmed. This was done by operating the journal at such low speeds that the film temperature rise was negligible, so that the lubricant viscosity could be taken as that for the ambient temperature. Data for curves of friction coefficient f *versus* $2\pi^2\mu N/P$, where μ is the lubricant viscosity, N the journal speed in r.p.s., and P the bearing load per unit projected bearing area, were obtained and plotted. The fact that the experimental points lay on a straight line passing through the origin was considered to represent a quantitative verification of the Reynolds theory. The slope of the line according to the Reynolds or Petroff theory should be r/c, where r is the journal radius and c the radial clearance. The slope of the line as obtained experimentally was 1550. Since the value of c is quite difficult of accurate determination by direct measurement, the process of determining r/c from the slope of the straight line of f *versus* $2\pi^2(\mu N/P)$, is actually the most accurate method of determining r/c from a practical point of view.

RESULTS

The transient history of the temperature rise, as observed in the present experiments, is shown by the curves of Figs. 3 and 4. The solid curves give the values of the temperatures as measured at the bearing surface by the thermocouple placed midway between the ends of the bearing and at its top. The dashed curves and solid points are the film temperatures as calculated theoretically. The data for Fig. 3 are those obtained with

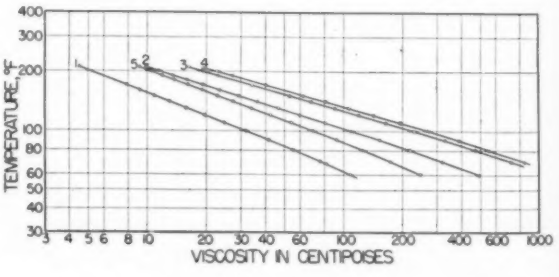

FIG. 2. Viscosity-temperature characteristics of lubricants used in the experiments. Curve 1: Gulflube 10; Curve 2: Gulflube 30; Curve 3: Gulflube 50; Curve 4: Gulflube 50+3% v.i. additive; Curve 5: Gulflube 10+10% v.i. additive.

[6] F. Morgan and M. Muskat, J. App. Phys. **9**, 539–46 (1938).

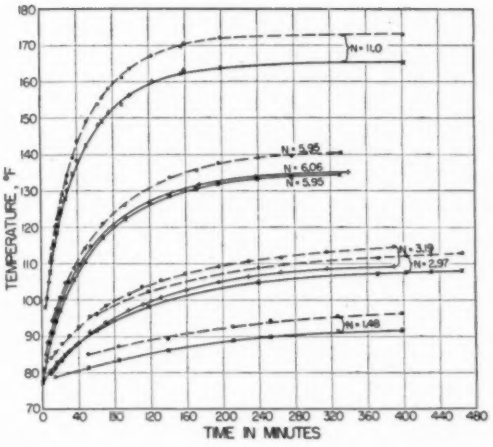

FIG. 3. Temperature variation of lubricant film as a function of time for Gulflube 10. Solid curves: temperature at bearing surface; Dashed curves: calculated film temperature; N = Journal speed in r.p.s.

Gulflube 10. Figure 4 gives the results when Gulflube 30 was the lubricant. Similar curves were obtained for Gulflube 50, Gulflube 50 plus 3 percent viscosity index additive, and Gulflube 10 plus 10 percent viscosity index additive. To avoid excessive overlapping of the curves, some of the series of runs were omitted from the plots of these figures.

As previously indicated, the theoretical temperatures were obtained by inverting the Petroff formula and calculating the effective film viscosity from the observed friction coefficients. In particular, the formula used was

$$\mu = fP/(2\pi^2 Nr/c). \qquad (1)$$

The use of this formula is based on the assumption that the journal bearing system was operating under thick film conditions and with no appreciable eccentricity. These assumptions, in turn, were amply justified by the linear plots of f *versus* $2\pi^2\mu N/P$, previously referred to, which showed that thick film conditions obtained even for values of the journal speed as low as 0.05 r.p.s., which were far lower than those used in the temperature measuring experiments. Combining the values of the viscosity, as calculated by Eq. (1), with the viscosity temperature characteristics of Fig. 2, the effective film temperatures were readily obtained.

As is to be expected, the effective film temperatures calculated theoretically are in all cases higher than the actual bearing temperatures. On the other hand, it is to be observed that the difference between the measured and calculated temperature is generally only a small fraction of the total temperature rises, except during the early part of the transient period. In fact, the difference was in all cases less than 8°F except for Gulflube 10 plus 10 percent viscosity index additive, for which case it amounted to 13.5°F. Thus from a practical point of view, it appears that at least in the thick film region the measurement of the bearing temperature should give a fair approximation to the average film temperature.

It should be noted that the calculated film temperatures reported here actually represent upper limits for these temperatures. For in the use of Eq. (1) no account was taken of the slight change in the radial clearance as the over-all temperature of the system is raised. Calculation shows that the composite bronze bearing and steel shell used in the experiments will have an effective thermal expansion coefficient 3.5 percent greater than the steel journal. This would imply an expansion in the radial clearance of less than 5 percent for a temperature rise of the whole system of even 75°C. Yet without this correction the resultant calculated values of film temperature would come out to be some

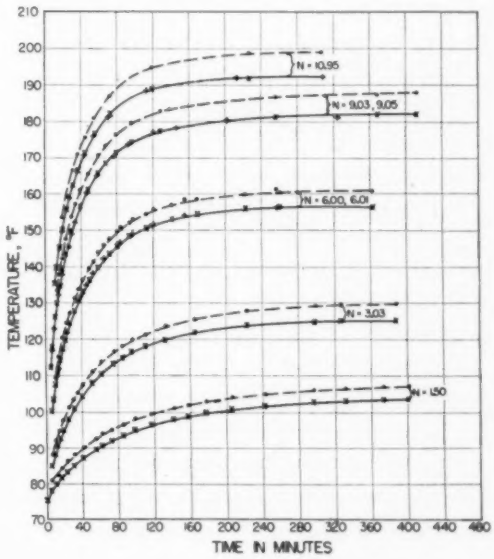

FIG. 4. Temperature variation of lubricant film as a function of time for Gulflube 30. Solid curves: temperature at bearing surface; Dashed curves: calculated film temperature; N = Journal speed in r.p.s.

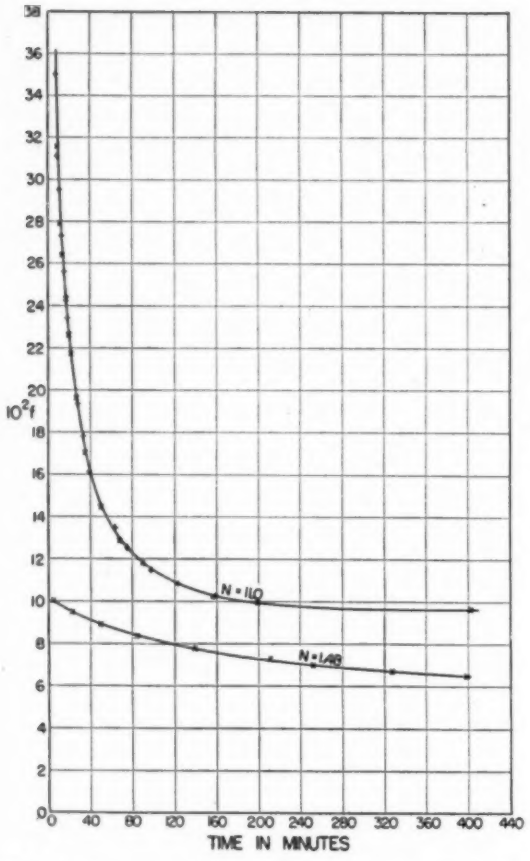

FIG. 5. The variation of the bearing friction coefficient f as a function of time for Gulflube 10. N = Journal speed in r.p.s.

2–4°F too high, which could account for an appreciable part of the difference between the bearing and film temperatures as reported here. In any case, the approximation of the film temperatures by the bearing surface temperatures in practical calculations is probably even better justified than would appear to be the case from a cursory inspection of the data given in Table I below.

The accuracy of the data such as are shown in Figs. 3 and 4 may be judged from the reproducibility of the results for repeat runs. The experimental points for repeat runs with the same journal speed are distinguished by the ×'s and +'s. Solid points for the calculated temperatures are also indicated for each set of repeat runs. It will be noted that the agreement between both the observed temperatures and those calculated for runs with the same journal speed is highly satisfactory.

The friction coefficient transients for Gulflube 10 and Gulflube 30 are shown graphically in Figs. 5 and 6. Only the two limiting curves for each type of lubricant have been plotted so as to avoid overlapping that would result from plotting of the data for the intermediate speeds. Except for the initial values of the friction coefficient, which are roughly proportional to the journal speed, the range in friction coefficients for different speeds is much less than that for the bearing or film temperatures. This, of course, is a consequence of the fact that at large values of the journal speed the friction coefficients are reduced more by the drop in lubricant viscosity resulting from the greater temperature elevations. It is to be noted that the drop in the film viscosity exactly parallels that in the coefficient of friction, as expressed quantitatively by Eq. (1).

Although the larger part of the ultimate temperature rise takes place during the first few

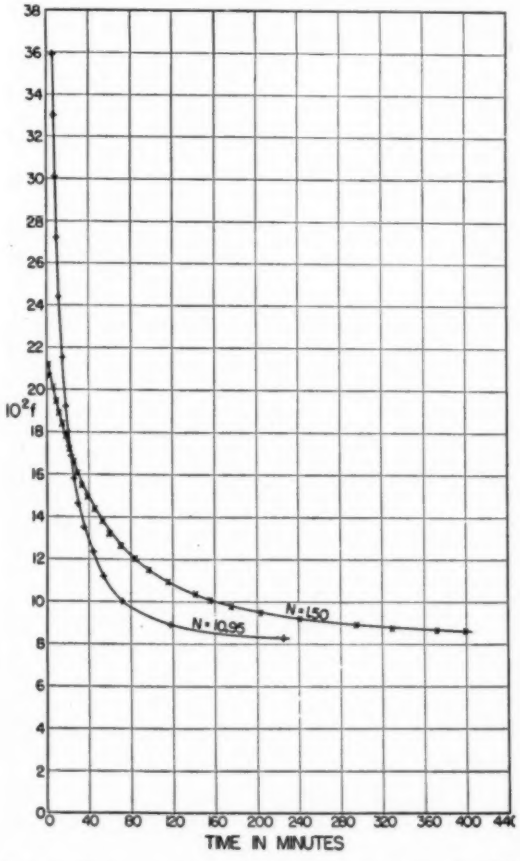

FIG. 6. The variation of the bearing friction coefficient f as a function of time for Gulflube 30. N = Journal speed in r.p.s.

hours of operation of the machine, the final equilibrium values frequently were not reached even after more than six hours of operation. To eliminate the uncertainty as to the time at which equilibrium may be assumed to have actually been reached, an attempt was made to derive the true asymptotic equilibrium values. This was based on the fact that, in general, equilibrium processes are approached asymptotically in an exponential manner. Accordingly, the temperatures were plotted on semi-logarithmic paper against the time, and the assumed ultimate equilibrium value was so varied that the tail end of the transient curves gave approximately straight lines. Such a procedure is clearly very sensitive to inaccuracies in the temperature data. Nevertheless, for practically all the runs made, it was possible to choose values for the equilibrium temperature which would give very satisfactory linear plots for the data taken near the time of completion of the experiments. Several typical plots are shown in Figs. 7 to 10. In these plots, the ordinate scales represent the numerical values of the differences between the equilibrium temperatures or friction coefficients and the instantaneous values, as indicated by the notations along the individual curves. The lower abscissa scales give the time after the beginnings of the runs for the friction and observed temperature curves. The upper abscissa scales give the corresponding times for the calculated temperature curves. These scales were shifted so as to avoid confusing overlapping among the curves. In some cases, the chosen values for the equilibrium temperatures turned out to be somewhat lower than the last values of temperature actually observed or calculated. While no positive explanation can be offered for such behavior, it is possible that in these cases the ambient temperatures may have shown abnormally rapid increases as the experiments were being concluded. It is also of interest to note in Figs. 7 to 10 that even the early parts of the transient histories connect rather smoothly to the linear asymptotic segments. This suggests that the composite transient behavior could be represented as a resultant of a small number of individual exponential terms. However, no detailed attempts were made to derive formulas representing the whole temperature histories.

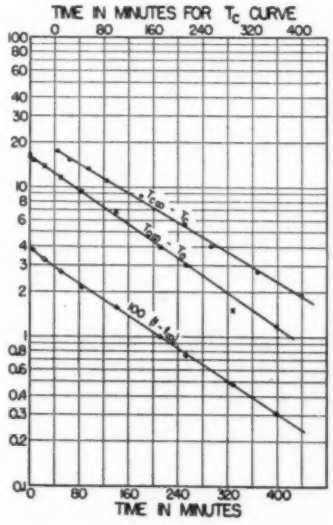

FIG. 7. Transient temperature and friction plots for Gulflube 10 and journal speed of 1.48 r.p.s. Initial temp.: 77.0°F; Final ambient temp.: 80.5°F; Equil. temp. (obs.): 92.8°F; Equil. temp. (calc.): 98.0°F; Equil. friction coeff.: 0.062; Load: 31,028 g.

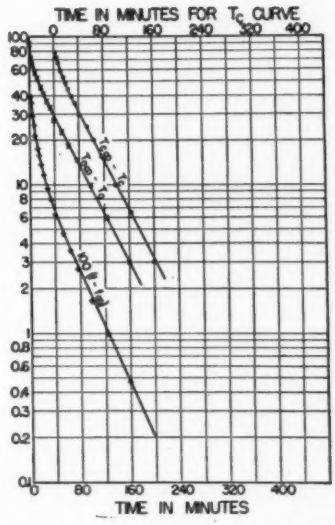

FIG. 8. Transient temperature and friction plots for Gulflube 10 and journal speed of 11.0 r.p.s. Initial temp.: 76.6°F; Equil. temp. (obs.): 166.0°F; Equil. temp. (calc.): 173.5°F; Equil. friction coeff.: 0.098; Load: 32,665 g.

In Figs. 7 to 10 are also plotted the calculated effective film temperatures, and the equilibrium values were also here so chosen as to make the latter parts of the curves linear. In many cases, this resulted in linear segments which were parallel to those for the observed bearing temperatures, thus indicating equal rates of approach

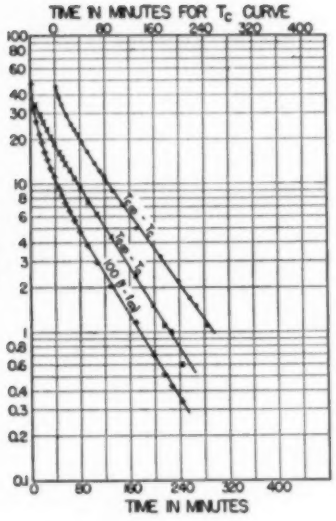

FIG. 9. Transient temperature and friction plots for Gulflube 30 and journal speed of 3.06 r.p.s. Initial temp.: 76.1°F; Final ambient temp.: 76.1°F; Equil. temp. (obs.): 123.6°F; Equil. temp. (calc.): 128.5°F; Equil. friction coeff.: 0.150; Load: 38,478 g.

to the equilibrium condition. In other cases, the slopes were appreciably different, although such differences may be mainly due to slight errors in the data which tend to accentuate the errors in the locations and slopes of the linear asymptotic segments.

In addition to the temperature data, the transient histories of the friction coefficient are plotted in Figs. 7 to 10. Here, too, the ultimate equilibrium values for the friction coefficient were chosen so as to give linear segments for the data near the completion of the runs. Again it is to be observed that the slopes of the friction coefficient linear segments are in many cases the same as for the temperature curves, and the discrepancies in others may here also be the result of slight errors in the data.

DISCUSSION

Equilibrium Temperature, Viscosity, and Frictional Force

The equilibrium data for the twenty-one separate runs which were made are summarized in tabular form in Table I. The equilibrium temperature elevations are plotted as a function of the journal speed in Fig. 11 for Gulflube 10 and 30. The ambient temperature has been arbitrarily taken as 3°F higher than the initial value for the five runs where only the latter was observed (marked by * in Table I). It will be noted that the temperature rise does not increase as rapidly as the speed when the latter exceeds 2 r.p.s. As is to be expected, the temperature rises are greater for the more viscous oil, Gulflube 30.

The fall in equilibrium viscosity with increasing journal speed is also shown in Fig. 11. Here the flattening in the curves is due not only to a decline in the rate of temperature rise, but also to the decreasing temperature sensitivity of the viscosity at the higher temperatures.

As the various runs were not all made with the same bearing load, no basic relation should be expected between the coefficient of friction and the journal speed. However, the frictional force, which determines the energy dissipated in the system, should show a significant relationship to the speed. The data for Gulflube 10 and Gulflube 30 are plotted in Fig. 12. It will be seen that for Gulflube 10, the frictional force increases monotonically with the speed at least up to 11 r.p.s. although the rate of rise quickly tapers off at speeds exceeding about 4 r.p.s. This deviation from non-linearity is evidently the consequence of the rising film temperature and falling viscosity as the speed is increased. In the case of Gulflube 30, the frictional force is a maximum at about 3 r.p.s. This is due to the higher viscosity

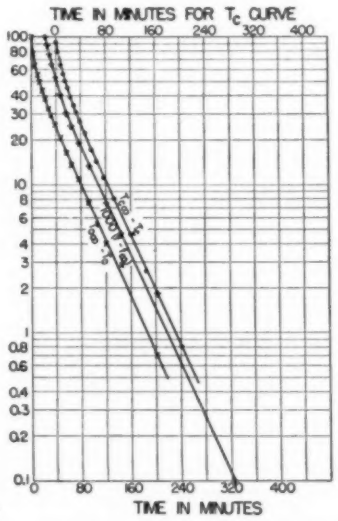

FIG. 10. Transient temperature and friction plots for Gulflube 30 and journal speed of 9.05 r.p.s. Initial temp.: 80.6°F; Final ambient temp.: 79.0°F; Equil. temp. (obs.): 181.0°F; Equil. temp. (calc.): 187.6°F; Equil. friction coeff.: 0.0812; Load: 60,175 g.

TABLE I.

Run No.	Lubricant	Speed N (r.p.s.)	Ambient temp., °F	Bearing load W (g)	Equil. temp. obs. °F	Equil. temp. calc. °F	Equil. viscosity obs. (cp)	Equil. viscosity calc. (cp)	Equil. friction coeff.	Equil. friction force (g)
1	Lube 10	1.48	80.5	31,028	92.8	98.0	38.7	33.3	0.0620	1924
2	Lube 10	2.97	81.0	31,028	108.5	113.0	25.3	23.0	0.0850	2637
3	Lube 10	3.19	80.7	31,028	110.5	115.0	24.3	22.0	0.0890	2762
4	Lube 10	5.95	81.5	31,028	134.7	141.0	14.5	13.0	0.0975	3025
5	Lube 10	6.06	80.0*	31,028	135.5	141.5	14.2	12.9	0.0970	3010
6	Lube 10	11.00	77.9*	33,214	165.3	173.5	8.4	7.5	0.0960	3189
7	Lube 10	11.00	76.6*	32,665	166.0	173.5	8.3	7.5	0.0980	3201
8	Lube 30	1.50	79.1	60,175	104.3	107.5	95.7	86.0	0.0840	5055
9	Lube 30	3.03	77.2	38,478	125.3	129.7	52.2	47.0	0.1440	5541
10	Lube 30	3.06	76.1	38,478	123.6	128.5	55.0	48.5	0.1500	5772
11	Lube 30	6.00	74.3*	38,478	156.7	160.8	25.2	23.1	0.1405	5406
12	Lube 30	6.01	74.9*	35,978	156.4	161.7	25.4	22.7	0.1485	5343
13	Lube 30	9.03	81.5	60,175	181.5	187.0	15.6	14.2	0.0829	4989
14	Lube 30	9.05	79.0	60,175	181.0	187.6	15.7	14.1	0.0812	4886
15	Lube 30	10.95	74.8	60,175	191.9	198.9	13.1	11.7	0.0830	4995
16	Lube 30	10.95	75.4	60,175	192.2	198.9	13.0	11.7	0.0825	4964
17	Gulflube 50	10.46	84.3	60,175	211.1	218.2	16.0	14.3	0.0973	5855
18	Gulflube 50	10.46	83.9	60,175	209.6	216.2	16.4	14.7	0.1000	6018
19	Gulflube 50+ 3% v.i. additive	10.46	85.0	60,175	214.4	221.4	17.9	15.7	0.1059	6373
20	Gulflube 50+ 3% v.i. additive	10.46	81.9	60,175	214.9	222.5	17.7	15.7	0.1058	6367
21	Gulflube 10+ 10% v.i. additive	10.96	80.2	33,213	185.7	199.2	12.7	10.6	0.1354	4497

* These are initial ambient temperatures. Final values were not observed.

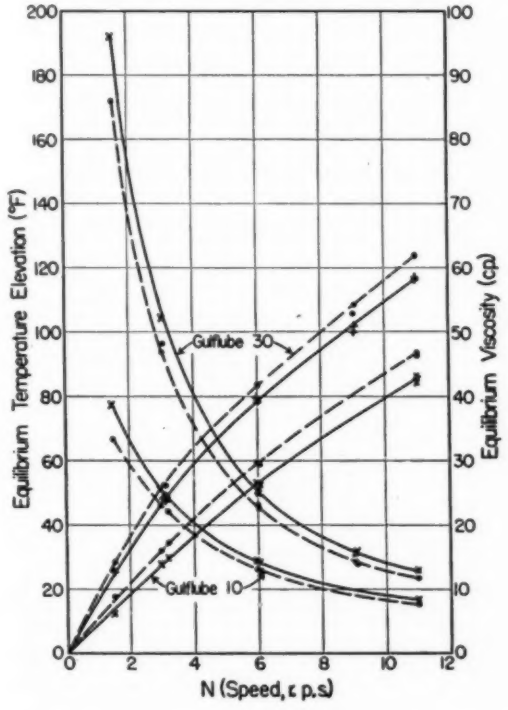

FIG. 11. The variation of the equilibrium temperature elevation and viscosity with journal speed N. Solid curves: directly observed; Dashed curves: calculated from friction measurements. Curves rising to right refer to temperature elevation; those rising to left refer to equilibrium viscosity.

temperature sensitivity of the Gulflube 30, and the fact that at the greater speeds the greater temperature elevation and viscosity reduction more than counterbalance the normal proportionality between frictional force and speed. This latter behavior is indicated by the dashed lines of Fig. 12, which correspond to constant lubricant viscosities equal to that at 80°F.

Heat Balance Correlations

Although the temperature elevation curves of Fig. 11 appear to show a systematic and regular behavior for the individual oils, it is reasonable to expect a relationship between the data for the different oils. For basically the equilibrium temperatures are determined by the heat transfer characteristics of the journal bearing systems as well as by the properties of the oil. Thus, by equating at equilibrium the frictional heat generation to the heat loss, no individual distinction need be made between the oils. Assuming the heat loss to be given by a power of the bearing temperature elevation, we have:

$$8\pi^3 r^2 (r/c) L \mu_\infty N^2 = k(T_\infty - T_a)^m, \qquad (2)$$

where μ_∞, T_∞ denote the equilibrium viscosity

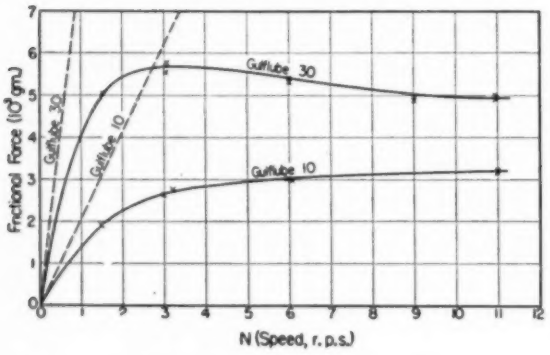

FIG. 12. Variation of the equilibrium frictional force with journal speed N. Solid curves: experimental data; Dashed curves: calculated on basis of constant film viscosity.

and temperature, T_a the ambient temperature, and k the over-all heat transfer constant. Hence $\mu_x N^2$ should be the same function of the temperature elevation, for all the lubricants, in a given journal bearing system. That this prediction is verified is shown by Fig. 13, from which it will be seen that the data for all the five lubricants studied fall on single straight lines within the experimental errors of the measurements, both for the observed and calculated temperature data.

The equations for the straight lines of Fig. 13 are:

$$\begin{aligned} \mu_x N^2 &= 2.55(T_x - T_a)^{1.35}, \quad \text{(observed)} \\ \mu_\infty N^2 &= 2.10(T_\infty - T_a)^{1.35}, \quad \text{(calculated)} \end{aligned} \tag{3}$$

where the term "observed" refers to the temperatures as actually measured at the bearing surface, and "calculated" implies that the viscosities and temperatures are calculated from the observed friction coefficients.

It is interesting, and perhaps quite significant, that the exponent of the temperature rise, i.e., the radiation exponent 1.35, agrees so closely with the value 1.3, quoted as the value found by Lasche in 1903.[2] In fact, this latter value appears to be the only one previously reported, except for that found by McKee,[7] namely, 1.65. Moreover, McKee's exponent refers to a particular operating condition for a single value of lubricant flow through the system.[8] Here, however, the lubricant flow was so small as to exert only a negligible effect on the heat balance. In any case, whether the agreement found here with Lasche's result is basically significant or purely accidental, Eq. (3) provides the key for the calculation of the equilibrium conditions even when the lubricant flow is a major factor in the heat removal.

The numerical coefficients in Eq. (3) are proportional to the effective heat transfer coefficients. In fact, on comparison of Eq. (3) with Eq. (2), it is readily found that $k = 2.72 \cdot 10^8$ or $k = 2.24 \cdot 10^8$ depending on the use of the first or second of Eqs. (3). These numerical values are based on the expression of lengths in cm and viscosity in cp. They do not represent basic physical constants, but rather empirical coefficients applying to the specific journal bearing system with which they were obtained. In terms of the heat transfer coefficient B used by Burwell,[9] which is essentially defined by: $B = FN/L\Delta T^{1.35}$, where F is the total frictional force, B has the value 2.0, where the heat energy is expressed as BTU/hr., the length in feet, and ΔT refers to the bearing temperature elevation. This may be compared with the value 2.9 assumed by Burwell, following Lasche.

On the basis of Eqs. (2) and (3), one can derive a quantitative correlation between the temperature elevation curves of Fig. 11 and the total

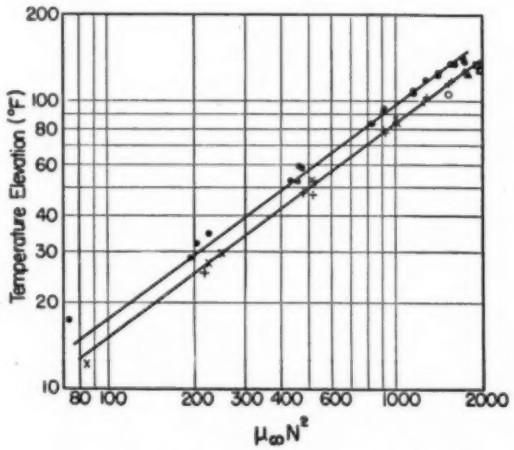

FIG. 13. Variation of equilibrium film temperature with $\mu_\infty N^2$. μ_∞: equilibrium viscosity (cp); N: speed (r.p.s.); ×: Gulflube 10; +: Gulflube 30; △: Gulflube 50; □: Gulflube 50+3% v.i. additive; ○: Gulflube 10+10% v.i. additive; ●: calculated film temperatures.

[7] S. A. McKee, J. Research Nat. Bur. Stand. **24**, 491–508 (1940).

[8] According to private communication from S. A. McKee.

[9] J. T. Burwell, Trans. A. S. M. E. **64**, 457–61 (1942).

frictional force curves of Fig. 12. Thus it is clear that the frictional force F will be given by:

$$F = \text{const. } \mu N = \text{const. } (\Delta T^m/N). \quad (4)$$

Hence:

$$\partial F/\partial N = \text{const. } (\Delta T^m/N^2) \times[-1+m(d \log \Delta T/d \log N)]. \quad (5)$$

Thus it follows that the equilibrium frictional force will increase with the speed where the slope of the log-log plot of ΔT *versus* N exceeds $1/m$, i.e., 0.74, and will decrease elsewhere. As a replot of Fig. 11 on a log-log scale shows the slope of the curve for Gulflube 10 is nowhere less than 0.74, a monotonic rise of F *versus* N is required as found in Fig. 12. On the other hand, the curve for Gulflube 30 has slopes exceeding 0.74 for $N<3$ and less than 0.74 for $N>3$. It is clear that the F *versus* N curve must then have a maximum, as is actually exhibited in the plot of Fig. 12.

The Rate of Approach to Equilibrium

The equilibrium heat balance condition of Eq. (2) may be applied to a prediction of the asymptotic rate of approach to the ultimate equilibrium state. Thus for the general temperature history of the system it is clear that the true variation of the bearing or film temperature will be given by:

$$C(\partial T/\partial t) = 8\pi^3 r^2 (r/c) L N^2 \mu - k(T-T_a)^m, \quad (6)$$

where C is the effective thermal capacity of the system.

It may be shown readily that the asymptotic solution of this equation is:

$$T_\infty - T = \text{const. } e^{-\alpha t}:$$

$$\alpha = \frac{k}{C}\left[\Delta T^m\left(\frac{m}{\Delta T} - \frac{\partial \log \mu}{\partial T}\right)\right]_\infty, \quad (7)$$

where $\Delta T = T - T_a$, and the bracket in Eq. (7) is to be evaluated for the equilibrium conditions.

From Fig. 2 it will be noted that at the higher temperatures,* the viscosity-temperature variation may be expressed by:

$$\mu = a/T^\beta. \quad (8)$$

* By a shift of the temperature scale the curves of Fig. 2 may be made linear throughout the whole temperature range of 70–210°F.

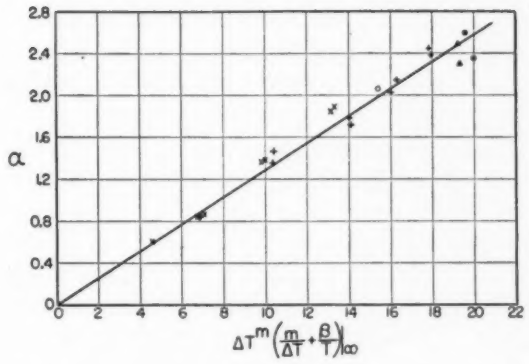

FIG. 14. The variation of the transient exponents α with the equilibrium temperature rise ΔT. T = equilibrium temperature; β = viscosity-temperature exponents; $m = 1.35$; ×: Gulflube 10: $\beta = 2.63$; +: Gulflube 30: $\beta = 3.25$; △: Gulflube 50: $\beta = 3.56$; □: Gulflube 50+3% v.i. additive: $\beta = 3.56$; ○: Gulflube 10+10% v.i. additive: $\beta = 2.74$.

Equation (7) then becomes:

$$\alpha = \frac{k}{C}\left[\Delta T^m\left(\frac{m}{\Delta T} + \frac{\beta}{T}\right)\right]_\infty. \quad (9)$$

For the oils used in the present study, β has the value 2.63 for Gulflube 10, 3.25 for Gulflube 30, 2.74 for Gulflube 10 plus 10 percent viscosity index additive, and 3.56 for Gulflube 50 and Gulflube 50 plus 3 percent viscosity index additive. As indicated by Fig. 13, m has the value 1.35.

In Fig. 14 are plotted the slopes α of the temperature lines such as shown in Figs. 7–10, against the bracket of Eq. (9), the plotted values representing in each case the means for the directly observed bearing temperatures and the calculated film temperatures. Since the choice of the linear segments of Figs. 7–10 may be materially affected by slight errors in the measured temperatures near equilibrium, Fig. 14 may be considered as a very satisfactory confirmation of Eq. (9).

Temperature Variation over Bearing Surface

The use throughout this study of the temperature at the top and center of the bearing as the index of the over-all bearing temperature was justified by direct measurements of the temperature distribution over the bearing surface. These were made by distributing four thermocouples over the central circumference of the bearing and four over a circumference one-fourth of the

bearing length from one end. Of the former, one was placed within the inlet opening for the lubricant and hence gave low readings. However, on comparing the remainder, it was found that while the averages of the equilibrium temperature readings over the central circumferential line were in all cases lower than those over the circumference near the end, the difference was in general only a fraction of a degree, and exceeded 1°F in only two runs. Moreover, there was a small variation along the central circumferential line, the thermocouples nearer the inlet, along the direction of rotation, giving somewhat lower readings. While these temperature variations are small, they are probably significant, and arise from the time lag in heating the lubricant as it drifts or flows from the central inlet to the open ends or other thermocouple stations. In fact, it should be emphasized that the over-all uniformity of the bearing temperatures observed here is probably due largely to the low rate of lubricant flow in the present experiments. Where the lubricant flow is sufficiently great appreciably to affect the equilibrium temperature conditions, it is to be expected that both the film and bearing temperatures will show a systematic rising trend in receding from the lubricant source which may well persist up to the exit boundaries. Under such circumstances, the equilibrium temperature conditions will not follow such simple relations as Eqs. (3), and the cooling effect of the lubricant will have to be taken into account. The way in which this may be done will be discussed elsewhere.

The authors are indebted to Mr. A. Chiazza and Mr. A. J. Schieritz for assistance with the experimental measurements, and to Dr. Paul D. Foote, Executive Vice President of Gulf Research & Development Company, for permission to publish this paper.

The Fundamental Problem of the Theory of Viscous Fluids

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A theorem is proved, giving the value of $d(\nabla\cdot\bar{v})/dt$ for viscous fluids. With the known value of $d\bar{\omega}/dt$, it gives a theoretical method of studying the fundamental problem of viscous fluids.

1.

IN this problem there are five unknown functions: v_x, v_y, v_z, components of the velocity vector $\bar{v}$ of a particle, the mean pressure p, and the density ρ at the point $\bar{e}$ (x, y, z) where the particle is located at the time t. The four independent variables are: x, y, z, t.

The viscosity μ, assumed constant, depends on the nature of the fluid, and the characteristic equation $F(p, \rho, T)=0$, is given by the nature of the flow; T is the temperature.

When ρ is a function of p only

$$\delta p/\rho=\delta f(p) \quad \text{where} \quad \delta=\nabla\cdot\delta\bar{e}.$$

2.

The fundamental problem may be stated as follows: given the distribution of $\bar{v}$, p, ρ, of all the fluid particles at a time t, determine the values of $\bar{v}$, p, ρ at subsequent times.

To solve the problem, we have five equations: (a) the three equations of Navier-Stokes

$$d\bar{v}/dt=\nabla\psi+(\nu/3)\nabla\theta+\nu\Delta\bar{v},$$

where

$$\nabla\psi=\bar{F}-\nabla p/\rho, \quad \nu=\mu/\rho, \quad \theta=\nabla\cdot\bar{v}$$

and $\bar{F}$ is the force at the considered point; (b) the equation of continuity $d\rho/dt=-\rho\theta$; (c) the characteristic equation.

The four first equations are relations between the partial derivatives of v_x, v_y, v_z, p, ρ, and with (c) can be solved in a few cases.

3.

However if at each point $\bar{e}$, where the particle is located at the time t, we consider the values of $\bar{\omega}=\nabla\times\bar{v}$ and $\theta=\nabla\cdot\bar{v}$, we can, by means of the Helmholtz equation, calculate the new values of $\bar{\omega}$ after an increment of time dt, namely, $\bar{\omega}+d\bar{\omega}$ and also the new values of θ, namely, $\theta+d\theta$ by means of a theorem that we shall prove hereafter. This theorem was proved for ideal fluids by Massau.[1]

The new values of ρ, namely, $\rho+d\rho$ will be calculated by means of the equation of continuity.

New values are calculated from the distribution of the initial values of $\bar{v}$ at the initial time t, a distribution which also gives the values of $\nabla\cdot\bar{v}$, $\Delta\bar{v}$, $\nabla\times\bar{v}$, $\nabla\cdot\Delta\bar{v}$, $\nabla\theta$, $\Delta\theta$; the distribution of $\bar{F}$, p, ρ gives $\nabla\nu$, $\Delta\psi$.

4.

Helmholtz' equation extended to viscous fluids is:[2]

$$d\bar{\omega}/dt=\tfrac{2}{3}\nabla\nu\times\nabla\theta-\nabla\nu\times\text{rot }\bar{\omega}+(\bar{\omega}\cdot\nabla)\bar{v}+\nu\nabla^2\bar{\omega}-\bar{\omega}\theta.$$

5.

Theorem giving $d\theta/dt$:

$$d\theta/dt=\Delta\psi+(\nu/3)\Delta\theta+\tfrac{1}{3}\nabla\nu\cdot\nabla\theta+\nu\nabla\cdot\Delta\bar{v}+\nabla\nu\cdot\Delta\bar{v}-\nabla_1\cdot[(\bar{v}_1\cdot\nabla)\bar{v}].$$

Proof:

$$(d/dt)\bar{v}=(\partial/\partial t)\bar{v}+(\bar{v}\cdot\nabla)\bar{v}. \qquad (a)$$

Applying the nabla operator

$$\nabla\cdot(d\bar{v}/dt)=\nabla\cdot(\partial\bar{v}/\partial t)+\nabla_1\cdot[(\bar{v}\cdot\nabla)\bar{v}_1]+\nabla_1\cdot[(v_1\cdot\nabla)\bar{v}]. \qquad (b)$$

We have also

$$(d/dt)\theta=(\partial/\partial t)\theta+(\bar{v}\cdot\nabla)\theta. \qquad (c)$$

Subtracting (b) from (c)

$$(d\theta/dt)-\nabla\cdot(d\bar{v}/dt)=-\nabla_1\cdot[(\bar{v}_1\cdot\nabla)\bar{v}].$$

Now we have the Navier-Stokes equation:

$$d\bar{v}/dt=\nabla\psi+\tfrac{1}{3}\nu\nabla\theta+\nu\Delta\bar{v}.$$

Operating by ∇.

$$\nabla\cdot(d\bar{v}/dt)=\Delta\psi+\tfrac{1}{3}\nu\Delta\theta+\tfrac{1}{3}\nabla\nu\cdot\nabla\theta+\nu\nabla\cdot\Delta\bar{v}+\nabla\nu\cdot\Delta\bar{v},$$

from which

$$d\theta/dt=\Delta\psi+(\nu/3)\Delta\theta+\tfrac{1}{3}\nabla\nu\cdot\nabla\theta+\nu\nabla\cdot\Delta\bar{v}+\nabla\nu\cdot\Delta\bar{v}-\nabla_1\cdot[(\bar{v}_1\cdot\nabla)\bar{v}].$$

In this equation we have:

$$-\nabla_1\cdot[(\bar{v}_1\cdot\nabla)\bar{v}]=2\Sigma\bar{k}\left(\frac{\partial\bar{v}}{\partial x}\times\frac{\partial\bar{v}}{\partial y}\right)-\theta^2,$$

where $\bar{i}$, $\bar{j}$, $\bar{k}$, are the unit vectors along the x, y, z axes assumed rectangular.

6.

The new values of $\bar{\omega}$, θ, and ρ, having been calculated for an increment of time dt, by integration can be found for any time. Since the new values of $\bar{\omega}$, θ are known at all points A in space, this enables us to calculate $\bar{v}$ at any point B in space by means of the known formula

$$\bar{v}=\int\frac{\theta}{4\pi}\frac{\bar{r}}{r^3}dV+\int\frac{\bar{\omega}}{2\pi}\times\frac{\bar{r}}{r^3}dV;$$

dV is the volume element at point A, $\bar{r}$ is the vector $\overline{AB}$; the integration is extended to all points A in space. Any additional velocity $\bar{w}=\nabla\varphi$ may be added to $\bar{v}$ if φ satisfies Laplace's equation $\Delta\varphi=0$.

[1] J. Massau, *Lectures on Mechanics* (Meyer, Ghent, 1896), p. 276.

[2] W. v. Ignatowsky, *Vector analysis* (Teubner, Berlin, 1910), Vol. II, p. 72.

The Viscosity of Dilute Solutions of Long-Chain Molecules

V. Dependence on the Solvent*†

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(Received March 1, 1943)

Staudinger's relation between η_{sp}/c and the molecular weight of a chain molecule solute should theoretically apply only if the chain molecules are kinked in a purely random way. With actual solutions, one would expect the kinked chains to be more or less tightly coiled than for purely random kinking, depending on whether the average cohesive energy density between *like* units (solvent molecules and solute submolecules) is greater or less than that between *unlike* units. The tighter the coiling, the less is the specific viscosity, for a given solute molecule chain length. This cohesive energy density difference is closely related to μ_1, the constant, characteristic of a given solute-solvent system, which enters into the equations recently derived by the author for the activities of the components and for related quantities, such as osmotic pressures, solubilities, and precipitabilities. From this relationship, it has been shown that the η_{sp}/c values for solutions of a given polymer in different solvents should, in general, be symbatic with the proportion of a poor solvent which is required to produce precipitation of a solid phase. Data from the literature on polystyrene and polybutene solutions are in agreement with this conclusion.

THE effect of the nature of the solvent on the viscosity of dilute solutions of long-chain molecules was briefly discussed in the third paper of this series:[1] "If the solvent molecules are strongly attracted by the atoms or groups all along the solute chain, they may, by steric hindrance, make the more coiled-up configurations impossible or improbable and so favor the more extended configurations. This increases the specific viscosity. . . . On the other hand, strong mutual attraction between solvent molecules, like strong attractions between different parts of a solute molecule chain, favor the coiled configurations and so low viscosities."

This same idea has been expressed by Flory[2] and by Alfrey, Bartovics, and Mark.[3] The last-named group of authors considered especially the effect of a change in temperature on the degree of coiling and so on the viscosity. They also presented some new experimental data verifying our (and their) qualitative conclusions.

In this paper, it will be shown that these considerations can be put on a semiquantitative basis.

For a solution of a very long-chain molecule, the ratio of specific viscosity to concentration, at infinite dilution, has been shown[4,5] to depend upon molecular weight according to the equation

$$\left(\frac{\eta_{sp}}{c}\right)_{c=0} = K_M M^{\nu}, \tag{1}$$

where ν equals 0, 1, or 2 if the chain molecules are coiled into a ball, kinked in a purely random manner, or extended into a rod-like configuration. (The K_M values need not be the same for these three cases.) It seems reasonable to assume that this relationship, with ν non-integral, also holds, approximately, for solutions of chain molecules which are kinked, but not in a purely random way. If so, the dependence of the limiting specific viscosity on molecular weight can be specified by the single constant ν.

From the qualitative argument just outlined, one may assume ν to be *antibatic** with a quantity

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* Communication No. 915 from the Kodak Research Laboratories.

† Presented before the Division of Paint, Varnish, and Plastics Chemistry of the American Chemical Society, at Buffalo, New York, September 8, 1942. Some of the material herein was previously presented at the Symposium on "Viscosity, Molecular Size, and Molecular Shape," held under the sponsorship of the Society of Rheology at the Polytechnic Institute of Brooklyn, February 20, 1942.

[1] M. L. Huggins, J. App. Phys. **10**, 700 (1939).

[2] P. J. Flory, J. Chem. Phys. **10**, 51 (1942).

[3] T. Alfrey, A. Bartovics, and H. Mark, J. Am. Chem. Soc. **64**, 1557 (1942).

[4] M. L. Huggins, J. Phys. Chem. **42**, 911 (1938).

[5] M. L. Huggins, J. Phys. Chem. **43**, 439 (1939).

* Two functions are mutually *antibatic* if an increase of the (algebraic) value of one is accompanied by a decrease in the (algebraic) value of the other. Two functions that are inversely proportional to each other are antibatic; the reverse, however, is not necessarily the case. Two functions are *symbatic* if an increase of the (algebraic) value of one is accompanied by an increase in the (algebraic) value of the other. They may or may not be directly proportional to each other.

TABLE I. Comparison of viscosities and solubilities of polybutene solutions in different solvents.

Solvent	η_{sp}/c at 25°C	Ml of 2-ethyl butyl alcohol added to 25 ml solution (conc. =10 g/l) to produce precipitation
Isoamyl butyrate	0.057	0.0
Isoamyl caproate	0.150	1.8
Benzene	0.221	14.6
Toluene	0.429	20.3
Xylene	0.678	21.7
n-Propylbenzene	0.690	21.7
Mesitylene	0.690	21.8
Cymene	0.714	21.9
Amylbenzene	0.784	22.0
n-Hexane	0.530	22.9
n-Heptane	0.855	26.4
n-Octane	0.923	26.5
Varsol No. 2	1.018	31.8

TABLE II. Comparison of viscosities and solubilities of polystyrene solutions in different solvents.

Solvent	η_{sp}/c at 20°C	Cc of precipitant added to 100 cc of 0.4% solution to produce precipitation	
		Methanol	Acetone
Ethyl acetate	4.6	9.2	75.2
Monomethyl ether of glycol acetate	4.9	4.2	63.2
Xylene	9.7	27.8	512
Toluene	10.1	30.6	520
Tetralin	10.4	—	602
Benzene	10.8	32.4	540
Chlorobenzene	10.9	36.0	622
Bromobenzene	10.9	37.0	708

A_{12}, defined by the relation

$$A_{12}=a_{11}+a_{22}-2a_{12}, \tag{2}$$

where a_{11}, a_{22}, and a_{12} are cohesive energy densities[6] between like solvent molecules, between like solute submolecules, and between unlike units (solvent molecules and solute submolecules, respectively). Strictly, a_{11}, a_{22}, and a_{12} should probably be defined as more complicated functions of the cohesive energy, involving the surface area and the shape and the distribution of attractive and repulsive forces in the units, as well as their volumes. Following Hildebrand[7,8] one may consider a_{12} and a_{22} as the "internal pressures" existing in the pure liquid solvent and solute, respectively, a_{12} being the internal pressure in a hypothetical liquid in which each solvent molecule is in contact only with solute submolecules, and vice versa.

A_{12} is not directly measurable. Scatchard and Hildebrand have shown, however, that the partial molal heats of mixing ($\Delta\overline{H}$) of the components of a solution should approximately obey the equations

$$\Delta\overline{H}_1=A_{12}\overline{V}_1V_2^2 \tag{3}$$

and

$$\Delta\overline{H}_2=A_{12}\overline{V}_2V_1^2, \tag{4}$$

in which $\overline{V}_1$ and $\overline{V}_2$ represent the partial molal volumes (equal, for our present purposes, to the molal volumes, V_1 and V_2), and V_1 and V_2 are the volume fractions. One may predict, therefore, that *the larger the ratio* $\Delta\overline{H}_1/\overline{V}_1$ (or $\Delta\overline{H}_2/\overline{V}_2$), *the smaller the limiting value of* η_{sp}/c (i.e., the smaller the value of ν). As far as the writer is aware, the requisite data for testing this prediction have not yet been obtained.

The solvent activity, a_1, in a solution of a long-chain compound depends on the volume fractions and partial molal volumes according to the relation[9-12]

$$\ln a_1=\ln V_1+\left(1-\frac{\overline{V}_1}{\overline{V}_2}\right)V_2+\mu_1V_2^2, \tag{5}$$

where

$$\mu_1=\epsilon_1+(A_{12}\overline{V}_1/RT). \tag{6}$$

The quantity ϵ_1 depends on the sizes, shapes, and flexibilities of the component molecules and also on the departure from perfect randomness of mixing and other factors.[13]

From Eq. (5), it follows that, *provided changes in* ϵ_1 *can be neglected*, $\mu_1/\overline{V}_1$ is symbatic with A_{12} and so antibatic with $(\eta_{sp}/c)_{c=0}$. Therefore, *in general, the larger the value of the ratio* $\mu_1/\overline{V}_1$, *the smaller is the viscosity*. Exceptions, resulting primarily from changes in the quantity ϵ_1, should, however, be expected.

The solubility (as expressed by the volume fraction V_2 of the solute in the saturated solution) of a solid long-chain compound in a small-

[6] G. Scatchard, Chem. Rev. **8**, 321 (1931).
[7] J. H. Hildebrand, J. Am. Chem. Soc. **57**, 866 (1935).
[8] J. H. Hildebrand, *Solubility of Non-Electrolytes* (Reinhold Publishing Corporation, New York, 1936).
[9] M. L. Huggins, J. Phys. Chem. **46**, 151 (1942).
[10] M. L. Huggins, Ann. N. Y. Acad. Sci. **43**, 1 (1942).
[11] P. J. Flory, reference 2.
[12] M. L. Huggins, J. Am. Chem. Soc. **64**, 1712 (1942).
[13] M. L. Huggins, Ann. N. Y. Acad. Sci. (in press).

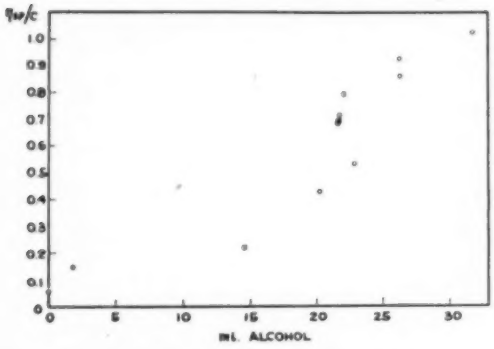

FIG. 1. Viscosities of polybutene solutions in various solvents at 25°C, plotted against the volume of 2-ethyl butyl alcohol which will produce turbidity when added to 25 ml of polybutene solutions (concentration, 10 g/l) in the same solvents. Data from Evans and Young (reference 14).

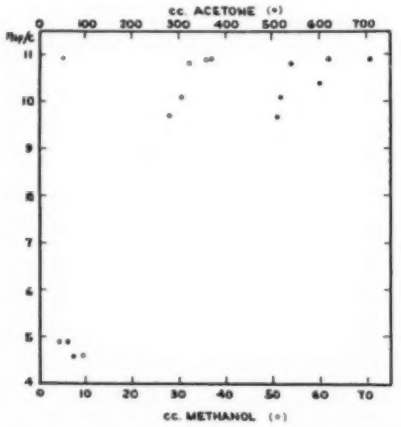

FIG. 2. Viscosities of polystyrene solutions in various solvents at 20°C, plotted against the volumes of methanol (○) and acetone (●) which will produce turbidity when added to 100 cc of polystyrene solution (concentration, 0.4%) in the same solvents. Data from Staudinger and Heuer (reference 15).

molecule liquid is related to $\mu_1/\overline{V}_1$ by the expression[12]

$$V_2=\exp\left(\frac{\overline{V}_2}{\overline{V}_1}V_1-V_1-\frac{\Delta F_{f,2}}{RT}-\overline{V}_2V_1{}^2\frac{\mu_1}{\overline{V}_1}\right), \quad (7)$$

in which $\Delta F_{f,2}$ is the free energy change on fusion of the solute at the temperature (T) of the experiment. We should expect this equation also to hold for a solution of a long-chain solute in a mixture of two small-molecule solvents, μ_1 and $\overline{V}_1$ representing some sort of average values. The addition of a poor solvent ($\mu_1/\overline{V}_1$ large) to a solution of a polymer in a good solvent ($\mu_1/\overline{V}_1$ small) obviously decreases the solubility V_2 (neglecting any change in the first term of the exponential, resulting from difference in $\overline{V}_1$ for the two solvents). If one adds the poor solvent to equal amounts of equal concentrations of solutions of the solute in various relatively good solvents, the larger the value of $\mu_1/\overline{V}_1$ for a given good solvent, the smaller is the amount of the poor solvent which must be added. (Complications in the quantitative interpretation arise if the precipitating phase is a gel, containing some of one or both solvents, rather than the pure solid solute, but qualitatively the same result would be expected.)

Thus, in general, the relative amount of poor solvent which must be added to a solution of a polymer in a better solvent to produce precipitation of a second phase is antibatic with the value of $\mu_1/\overline{V}_1$ in this solution, while this ratio, in turn, is antibatic with the value of $(\eta_{sp}/c)_{c=0}$ for this solution. Therefore, the amount of poor solvent required for precipitation should, as a rule, be symbatic with $(\eta_{sp}/c)_{c=0}$.

The data in Tables I and II and Figs. 1 and 2 show that this is the case for polybutene[14] and polystyrene[15] solutions in various solvents—provided one makes the reasonable assumption that the order of $(\eta_{sp}/c)_{c=0}$ values for the different solvents is the same as the order of η_{sp}/c values at the concentration used for the viscosity measurements. The minor discrepancies probably result, in part, from differences in the molal volumes of the solvents, and in part from differences in the magnitude of ϵ_1 (Eq. (6)).

[14] H. C. Evans and D. W. Young, Ind. Eng. Chem. **34**, 461 (1942).
[15] H. Staudinger and W. Heuer, Zeits. f. physik. Chemie **A171**, 129 (1934).

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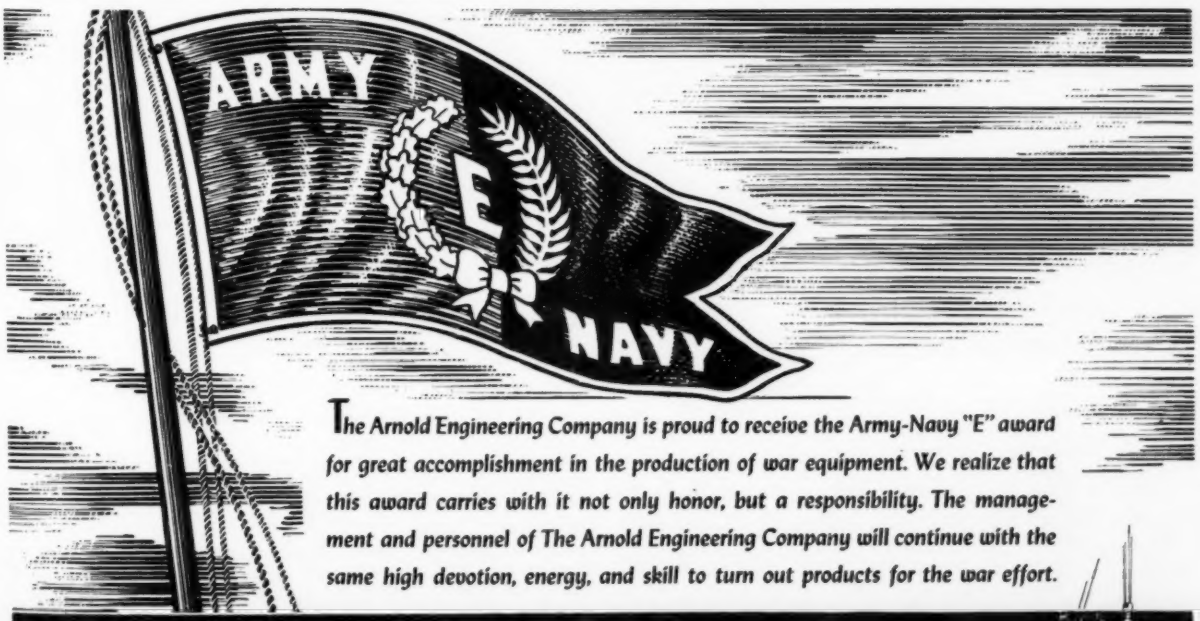

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Recent Applications of Physics

Small Steel Mill Plays Unexpected Role in Plane Production

A small steel mill in Pittsburgh is playing an unexpected role in plane production. Used by Howard Scott and William Johnson, Westinghouse research metallurgists, to replace a thermometer maker's dwindling supply of a special metal alloy, it has averted a threatened break in production of gauges.

Originally developed at Westinghouse as a metal sealer for electronic tubes, the alloy known as Kovar was then adopted to measure heat in plane engines and wings by electrical resistance.

War demands depleted the thermometer maker's original supply, which could not be replaced. Then the metallurgists went to work, painstakingly measuring the alloy ingredients and carefully cooking the mixture in a tiny electric furnace. The research men finally duplicated the original supply of the special alloy, so that plane-thermometer output can keep pace with the Air Forces' demands.

★

New Power-Recovery Method

More than half the power needed to operate an engine factory can be recovered from tests of plane engines, G. E. Cassidy, W. A. Mosteller, and W. L. Wright of the General Electric Company reported to the meeting of the American Institute of Electrical Engineers. The power-recovery method has helped the war effort by giving to the aircraft industry an improved testing technique. Using an induction generator, one of the latest set-ups begins the test with a cold start and run-in test for checking mechanical operation and lubrication of the engine. Then speed is gradually increased. When ready to "fire" the ignition switch is turned on and the engine throttle adjusted to idling speed. As the throttle is opened, the generator speed passes through synchronism and load is automatically applied to the engine. Any desired values of load and speed can be established. From such a test, the engineers reported that 3,000 to 6,000 kilowatt-hours of energy may be recovered from each engine of current-large rating.

★

Air-Cooled Gloves for Airdrome Workers

Air-cooled gloves now protect the hands of workers and improve the manufacture of landing lights for airdromes. A low pressure air hose, extending into the gauntlet of each asbestos glove, banishes the possibility of burns to workmen handling hot landing lights and accelerates the work. Reduction of glass breakage due to "shrinkage" when lamps are transferred from preheating temperature to sealing temperature is another advantage. The added protection permits the use of higher temperatures in the preheating process.

X-Ray Tubes with Beryllium "Windows"

Windows made of the metal beryllium, which though opaque to light are more transparent to certain kinds of x-rays than glass, are speeding tests of material used in bombing planes and other war machines. In such applications the x-rays are used to study the way the atoms are arranged. The lattice of atoms that make up the crystals of the metal diffract the x-ray beam. When it falls on a photographic plate there is no longer the single spot which there would be if the metal were not in the path. A symmetrical pattern of spots appears.

These x-ray diffraction patterns tell the scientist important facts about the molecular structure of the material being examined. He can tell how the molecules are arranged and the distances between them. In the case of various alloys, for example, it is possible to check the effect of heat treatment which changes the structure to make the metal stronger and more durable.

According to Zed J. Atlee, of the General Electric X-Ray Corporation, beryllium is ideal for use in windows transmitting long wave x-rays, when prepared by an electrolytic process. Its use has permitted other improvements in the design of x-ray equipment for diffraction studies. The thin beryllium disk is soldered to a ring of Fernico, and this can be fastened to hard glass, permitting the construction of a smaller and more efficient tube than the older one employing Lindemann glass which could only be fastened to soft glass. The greater x-ray transparency of the beryllium, and the improved design, have increased by some 15 times the speed with which diffraction patterns are obtained. Though beryllium, which is sometimes called glucinum, has previously been used for alloying with copper and other metals, this is the first use ever found for it in pure form. Produced by General Electric Company, Schenectady, New York.

★

New Contact Lights for Air-Fields

Airport runway marker lights, called contact lights, indicate to the pilot the point of contact with the ground. A new design had to be developed to accommodate the big bombers and stratoliners which would crash the button lighting units in use for smaller craft.

The new construction utilizes the principle of bridge design, with truss sections and no cantilever members. Strategic materials had to be avoided; rubber gaskets have been eliminated; shock-resisting glass and pre-focused lamps have been added. The two-element lens system has been replaced with a single lens, thereby obtaining an eight percent increase in light efficiency. All manufacturers of this new light use the same standard mounting dimensions and the sub-assemblies made by any company will fit all other units. This fact makes the new contact light advantageous for use on military airports here and abroad.

GEOPHYSICS

Volume VIII, Number I
(January, 1943)

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An Analysis of Abnormal Reflexions — *L. E. Deacon*

A Resistivity Survey of the Monument Oil Field — *C. M. England*

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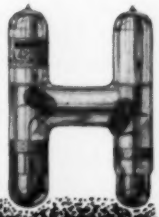

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Ultraviolet Rays Used to Destroy Virus

Dr. Harvey C. Rentschler, Director of Research at the Westinghouse Lamp Division, recently reported experiments with ultraviolet light as a means of destroying disease-carrying virus. Influenza, infantile paralysis, and the common cold are some of the diseases thought to be caused by virus which are so small that they pass through all filters and can only be seen for a moment in outline by the powerful electron microscope. Virus may be spread by air, as a result of coughing or sneezing.

For the past two years Dr. Rentschler, with the assistance of Miss Galina Mouromseff, staff bacteriologist, has been conducting thousands of tests using the germ-killing Sterilamp in an attempt to determine the exact amount of ultraviolet radiation required to inactivate virus in various quantities and conditions. As a result of these experiments it has been determined that photons of ultraviolet light must strike a vital spot in the virus molecule or else the virus is unharmed. Consistent 100 percent destruction is yet to be achieved, due to this difficulty. Data are being compiled on the amount of ultraviolet required to kill a given quantity of virus so that Sterilamps can be installed with greatest effect in schools, homes, offices, and factories.

★

New Welding Method

American warplanes can now be put together at speeds as fast as 1800 welding "stitches" a minute, with the aid of electronic tubes much like those in an ordinary radio set, Edwin H. Vedder, a Westinghouse engineer, reported recently at a meeting of the Pittsburgh section of the American Institute of Electrical Engineers.

Peak production in airplane plants, according to Mr. Vedder, depends upon resistance welding done by machines which have water-cooled copper jaws that clamp down to hold together the two pieces of metal being welded. When electricity passes through the jaws and into the metal, heat is created to weld the two inner surfaces tightly together. On some jobs the jaws are replaced by copper wheels that roll over the metal, making a continuous seam between the two pieces. The length of the time the electric current flows must be regulated precisely—sometimes to within less than a hundredth of a second. Such division of time is measured out by electronic tubes. Both in speed and efficiency this method is an improvement over previous methods used.

★

New Electrical Insulation

A new line of electrical insulation known as Multiform Insulators has been developed to supplement the supply at a time when war requirements are straining existing facilities. The Multiform process has overcome previous drawbacks in conventional glass-making methods. Through the new method, electrical glass is now available for insulation in a wide variety of shapes. The Multiform process involves a combination of cold-molding batch materials and subsequent fusing. Finished ware, in contrast with the more familiar types of glassware, is opaque or translucent; but it is a true glass. Products made by this process run the gamut from small insulating beads, several thousand to the pound, to large insulators weighing twenty-five pounds or more. Countersunk and tapped holes are practicable, as are both external and internal threads and grooves. These products can be made accurately and general dimensional tolerances are: large or heavy pieces, intricate shapes, hollow cylindrical sections—plus or minus 2.0 percent or 0.010"; flat plates, solid rods, disks, beads, bushings—plus or minus 1.0 percent or 0.005", excepting thickness which should be plus or minus 4.0 percent or 0.005". Also, this glassware can be ground and polished to closer tolerances if necessary. Seals to other appropriate glasses can be made. The Multiform Glassware can be made from almost any glass composition. This glass meets all the requirements of U. S. Navy Standard RE-13A-317F, Grade G, and is suitable for a wide variety of electric applications. Laboratory tests have shown that all the above glasses and others will meet the proposed A.S.A. American War Standard on Radio Insulating Materials of Low Dielectric Constant, Grades L-5 and L-6. The Multiform process is the product of the Corning Glass Works, Corning, New York.

★

New Insecticide Protects Soldiers Against Disease

A liquid insecticide developed by Dr. Lyle D. Goodhue, a young Department of Agriculture chemist, is protecting America's fighting men in tropical jungles against malaria and yellow fever. The insecticide is released as a fine mist that remains suspended in space and exterminates disease-carrying insects while remaining harmless to humans. The six-inch metal dispensers—each about the size of a tin can—are called "bombs" by workmen who make them at the rate of thousands a day in an eastern Westinghouse plant. The "health bombs" will be discharged frequently in tents and barracks wherever troops are stationed in the tropics, and in the cabin of every airplane—military and civilian—that takes off from a tropical base. Each dispenser is loaded with one pound of the liquid insecticide. In the 12 to 14 minutes required for complete discharge, one dispenser will fumigate 150,000 cubic feet of space, the equivalent of 240 army pup tents, or 50 giant bombers. But the spray can be turned on and off as necessary. For example, only three seconds are required to fumigate one pup tent. The insecticide is effective against bugs and insects, including flies, mosquitos, ants, and cockroaches. With the dispenser, the Army hopes to reduce sharply the casualty rate of past wars, in which disease incapacitated as many men as did bullets. Westinghouse Electric, East Pittsburgh, Pennsylvania is manufacturing the dispensers at a daily rate exceeding 10,000.

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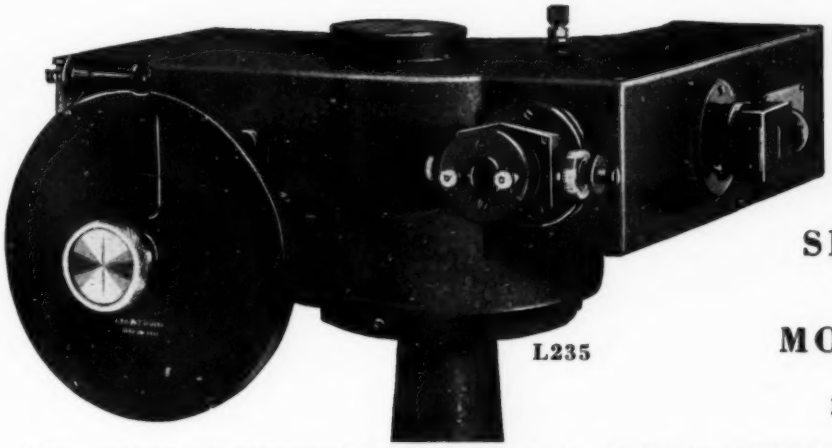

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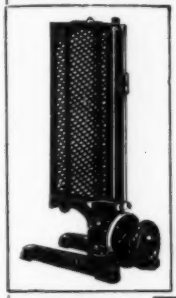

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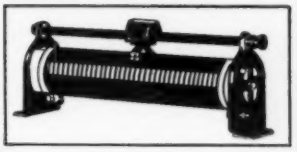

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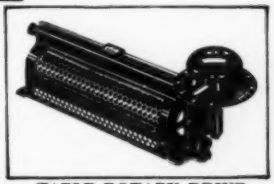

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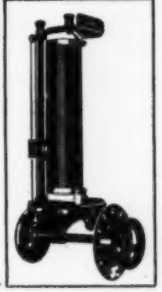

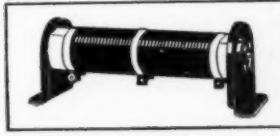

ADJUSTABLE RESISTORS with and without protective housing.

Nature of Lightning

Charles F. Wagner, speaking to the Academy of Science and Art of Pittsburgh in Carnegie Music Hall recently, pointed out that lightning—contrary to the popular conception of a single downward stroke—is composed of two separate strokes. The first is a downward flash, at the rate of 100 miles per second, with a light so feeble that it often fails to make an impression on photographic film. The visibly brilliant upward stroke moves along a highway of ionized air, established by the earlier stroke from cloud to ground, at a rate of 20,000 miles per second. The few ten-thousandths of a second separating the two strokes causes the human eye to see them as one.

★

New Substitute for Rubber

A semi-plastic material has been pressed into service to release large quantities of rubber heretofore required to operate intricate machines used to build RCA radio tubes. After experimentation "resistoflex," a semi-plastic made of polyvinyl alcohol, was found to be highly resistant to the many solvents that depreciated the rubber and ruled out the use of other plastics. RCA and Resistoflex Corporation engineers worked out formulas for suitable material to obtain the quality of flexibility that was required.

Under actual operating conditions, the new rubber-less hoses made from this material have stood up under months of hard use even though saturated with hot oil and subjected to continuous flexing at the rate of 600 times an hour. An application placed on one machine a year ago has shown no signs of depreciation, although the rubber tubing formerly used had to be replaced about five times annually.

The new tubing has eliminated, in great part, the use of hose made from strategic materials now critically needed for the war effort. The material also has been used to replace rubber parts other than hoses such as: washers, spacers, rollers, and other machine parts. In these mechanical applications, the material has consistently outlasted rubber. Product of RCA Manufacturing Company, Inc., Camden, New Jersey.

New Instrument Booklets

The Aerovox Research Worker, August, 1942, Vol. 14, No. 8, features "Capacitors in control circuits." Copies of this publication may be obtained from Aerovox Corporation, New Bedford, Massachusetts and subscription rates are 50 cents per year in U. S. A. and 60 cents in Canada.

The Aerovox Research Worker, September-October, 1942, Vol. 14, Nos. 9 and 10, presents an article on "Design data for *m*-derived type filters, Part I." Copies are obtainable from Aerovox Corporation, New Bedford, Massachusetts under their rates of 50 cents per year.

Bakelite Review, January, 1943, Vol. 14, No. 4, features an article on "The characteristics and uses of elastic vinyl plastics," describing replacement of rubber for calendering, extruding, and molding by these new plastics. Other articles are: "Machining laminated plastics efficiently," "Industrial camouflage with coatings based on Bakelite C-9 resins," and "New ideas on the 'home' front," which describes how plastics speed construction in wartime housing projects, and "Plastics that glow." Quarterly publication of Bakelite Corporation and the Plastics Division of Union Carbide and Carbon Chemicals Corporation, 30 East 42nd Street, New York, New York (23 pages).

Eyes for Industry defines and explains types of stroboscopes and their operating techniques. This occasional publication may be obtained from General Radio Company, Cambridge, Massachusetts (25 pages).

The General Radio Experimenter, December, 1942, Vol. 17, No. 7, presents an article on "Voltage regulation of variacs and effects of substitute materials." This booklet may be obtained from the General Radio Company, 30 State Street, Cambridge A, Massachusetts (7 pages).

Hardness is a booklet containing "A critical examination of hardness, dynamic hardness, and an attempt to reduce hardness to dimensional analysis." It has been shown by P. Roudie that by adopting a dynamic method of hardness testing he could as a consequence define hardness as a quantity of energy, and, therefore, as a function of the three fundamental units of mass, length, and time. The booklet describes Roudie's idea in extenso. The Nitralloy Corporation believes that a study of dynamic hardness will, once the principle is thoroughly understood, yield important advances in engineering. This is the reason why it has taken upon itself the duty of publishing the work. Another reason is that of bringing to the American engineers the writings of a Frenchman whose work has so far remained, if not unknown, at least unnoticed since its publication in 1930. This booklet may be obtained from The Nitralloy Corporation, 230 Park Avenue, New York, New York (105 pages).

Inco Nickel Alloys for the Process Industries is a bulletin presenting brief but comprehensive data on the eight Inco high nickel alloys and their corrosion resistant uses in the many chemical process industries. The individualized service characteristics of the metals, their average mechanical properties, physical constants, and other pertinent data are presented in comparative tables and charts which give "at a glance" information. This booklet may be obtained by addressing The International Nickel Company, Inc., 67 Wall Street, New York, New York (16 pages).

Modern Precision, Autumn, 1942, Vol. 2, No. 2, presents articles on process, ceramics, and metals industries as well

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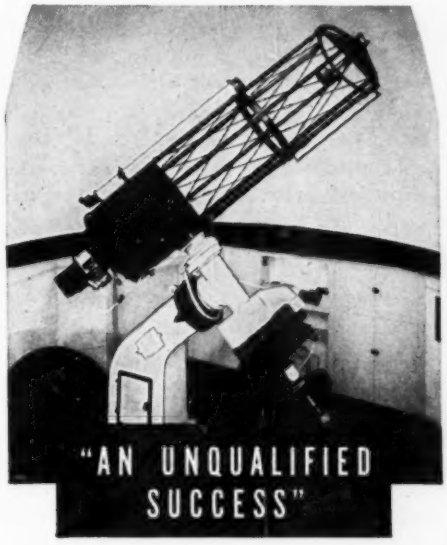

From Volume VIII, No. 6, Publications of The Observatory of The University of Michigan describing the Francis C. McMath Memorial 24-inch Reflecting Telescope, now in operation at the McMath-Hulbert Observatory of the University of Michigan.

"The mirrors were entrusted to The Perkin-Elmer Corporation. Our specifications were exacting, as no part of any surface could depart from the theoretical surface by more than one-tenth of a standard wavelength... Perkin-Elmer Corporation completed the primary mirror by conventional methods... It was then tested, pronounced well within the specifications, and found to have an unusually fine surface. The two high magnification secondaries, however, presented real difficulties. McCarthy, of Perkin-Elmer, felt that conventional methods of testing were inadequate and proposed a new method, which he has recently described. The Perkin-Elmer Corporation made up the necessary auxiliary optical equipment, and our two secondaries were figured by the new method, which eliminates the combined testing of primary and secondary. These two mirrors have been an unqualified success, both focal lengths being well within specifications, while the figuring is superb. Exposures for the disk of Jupiter are shorter by a factor of at least twelve when compared with our old 10½-inch telescope."

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as on the oil industry and power plants. Publication of the Leeds & Northrup Company, 4908 Stenton Avenue, Philadelphia, Pennsylvania (16 pages).

New Parts for Old describes four different methods for renewing worn metal parts such as pump rods and shafts. Using a worn pump shaft as a practical example, the booklet explains and illustrates the step-by-step procedures for reconditioning shafts. Occasional publication of The International Nickel Company, Inc., 67 Wall Street, New York, New York (7 pages).

Nitriding Furnaces, recently published, is divided into four captions or parts as follows: Part I. Nitriding and Nitriding Furnaces; Part II. Ammonia and Its Handling; Part III. Determination of Furnace Size for Nitriding; Part IV. Instruments for Measuring Ammonia Flow, etc. To those who contemplate a nitriding furnace installation this book will give worth while facts of importance. For others who may not have enough work to warrant an installation, a list of nitriding agents is given at the end of the book. Published by The Nitralloy Corporation, 230 Park Avenue, New York, New York (99 pages).

The Ohmite News, March, 1943, presents in news bulletin form an article on the Ohm's Law Calculator. Published by Ohmite Manufacturing Company, 4835–41 Flournoy Street, Chicago, Illinois (2 pages).

RCA Guide for Transmitting Tubes is a new and completely revised edition, designed especially for radio engineers and technicians in the armed service and war industries. Written originally and carried through in the three earlier editions as a catalog for tube users, the present or fourth edition of the Guide transforms the booklet into an effective instrument of war. By circuit designs, photographs, and technical details it supplies a wealth of information invaluable to every engineer and experimenter. For copies of this booklet write to Julius Haber, Press Department, RCA, Camden, New Jersey (72 pages).

RCA Tube Picture Book is prepared especially for use in radio war training centers. This booklet provides visual instruction in the details of various types of vacuum tubes. It contains eight charts showing structural details of representative receiving, transmitting, cathode-ray, and special tubes. Copies may be obtained for 10 cents each from the Commercial Engineering Section, RCA Victor Division, Radio Corporation of America, Harrison, New Jersey (16 pages).

Tips on Making Transmitting Tubes Last Longer is a new booklet designed to aid all users of electronic tubes in the industrial field as well as broadcasters. It describes how radio tubes wear out sooner when they are operated at maximum voltage capacity. Detailed instructions are given for the right method of putting tubes into operation by a slow start, explaining what happens to curtail the life of a tube when the full current is turned on at once. Five general "good rules to follow" are listed: a chapter on how to double the life of hard-to-get tungsten filament tubes, another on six ways to make mercury-vapor rectifier tubes last longer, one on tube resting periods, and one explaining why cooler tubes last longer. Single copies may be obtained from RCA Commercial Engineering Section, Harrison, New Jersey (11 pages).